PERMUTATION PATTERNS 2012

Monday, June 11

08:00 – 08:50 Registration

08:50 – 09:00 Conference opening

Morning session chaired by Anders Claesson

09:00 – 09:30 **Sergey Kitaev**
On permutation boxed mesh patterns

09:30 – 10:00 **Andrew M. Baxter**
Shape-Wilf-equivalences for vincular patterns

10:00 – 10:30 Refreshments

10:30 – 11:00 **Vincent Vajnovszki**
Lehmer code transforms and Mahonian statistics on permutations

11:00 – 11:30 **Mark Dukes**
Parallelogram polyominoes, the sandpile model on $K_{m,n}$, and a q,t-Narayana polynomial

11:30 – 12:00 **Hjalti Magnússon**
Stack-sorting and preimages of mesh patterns

12:00 – 13:30 Lunch

Afternoon session chaired by Lara Pudwell

13:30 – 14:00 **Rebecca Smith**
Sorting with modified devices

14:00 – 14:30 **Dominique Rossin**
Asymptotics of push-all permutations

14:30 – 15:00 **Adeline Pierrot**
Two-stacks sorting is polynomial

15:00 – 15:30 Refreshments

15:30 – 16:00 **Mathilde Bouvel**
Enumeration of permutations sorted with two passes through a stack and D_8 symmetries

16:00 – 16:30 **Janine LoBue**
Permuted Basement Fillings, k-ary Trees, and Watermelons

Tuesday, June 12

Morning session chaired by Mike Atkinson

09:00 – 09:30	**Brian Nakamura** Permutations with exactly r occurrences of a length three pattern
09:30 – 10:00	**Alexander Woo** Bruhat graphs and pattern avoidance
10:00 – 10:30	Refreshments
10:30 – 11:30	**Vincent Vatter** Small permutation classes
11:30 – 12:00	**Miklós Bóna** Surprising Symmetries in Objects Counted by the Catalan numbers
12:00 – 13:30	Lunch

Afternoon session chaired by Mathilde Bouvel

13:30 – 14:00	**Cheyne Homberger** Expected Patterns in Permutations Avoiding 123
14:00 – 14:30	**Vít Jelínek** Stanley-Wilf limits of layered patterns
14:30 – 15:00	**Alexander Burstein** A combinatorial proof of joint equidistribution of certain pairs of permutation statistics
15:00 – 15:30	Refreshments
15:30 – 16:00	**Anant Godbole** Covering all n-permutations with $(n+1)$-permutations
16:00 – 16:30	**Martha Liendo** Random Superpatterns

Wednesday, June 13

Morning session chaired by Rebecca Smith

09:00 – 09:30	**Mike Atkinson** Priority Queues and Pattern Classes
09:30 – 10:00	**Michael Albert** PermLab: software for permutation patterns
10:00 – 10:30	Refreshments
10:30 – 11:00	**Ruth Hoffmann** PatternClass: A GAP Package for Permutation Pattern Classes
11:00 – 11:30	**Marie-Louise Bruner** A Fast Algorithm for Permutation Pattern Matching Based on Alternating Runs
11:30 – 12:00	**Henning Ulfarsson** Automated discovery of permutation patterns
12:00 – 13:30	Lunch
13:45 – 17:00	Excursion to Stirling Castle
18:00 – 20:00	Conference dinner at The River House Restaurant

Thursday, June 14

Morning session chaired by Sergey Kitaev

09:00 – 09:30 **Jeffrey Remmel**
Up-down ascent sequences and the q-Genocchi numbers

09:30 – 10:00 **Miles Jones**
Generating functions for permutations with no consecutive pattern matches within the cycles

10:00 – 10:30 Refreshments

10:30 – 11:00 **Adrian Duane**
Consecutive Patterns in up-down permutations

11:00 – 11:30 **Luca Ferrari**
The Möbius function of the consecutive pattern poset

11:30 – 12:00 **Mark Tiefenbruck**
Extending from bijections between marked occurrences of patterns to all occurrences of patterns

12:00 – 13:30 Lunch

Afternoon session chaired by Einar Steingrimsson

13:30 – 14:00 **Brian Miceli**
Generalized Interval Embeddings

14:00 – 14:30 **Sen-Peng Eu**
Adin-Roichman-Mansour type identities

14:30 – 15:00 **Kassie Archer**
Periodic patterns of k-shifts

15:00 – 15:30 Refreshments

15:30 – 16:15 Problem session

16:15 – 17:00 Business meeting

Friday, June 15

Morning session chaired by Mark Dukes

09:00 – 09:30 **Lara K. Pudwell**
Non-contiguous pattern avoidance in binary trees

09:30 – 10:00 **Adam M. Goyt**
Pattern Avoidance in Ordered Partitions

10:00 – 10:30 Refreshments

10:30 – 11:00 **Benjamin Fineman**
Bounds for the number of permutations containing a low density of patterns

11:00 – 11:30 **Jennie Hansen**
Random permutations (and beyond)

PermLab: software for permutation patterns

Michael Albert (University of Otago)

PermLab is a Java software suite for assisting in research about (mostly classical) permutation patterns. It consists of a GUI which greatly extends the capabilities of its predecessor *Class-Counter* to include both graphical display and manipulation of single permutations, and the display of animations which can reveal underlying structure within collections of permutations. Additionally it includes an extensive applications programming interface which permits the development of special purpose code to deal with specific problems without requiring *too* much familiarity with Java. As well as demonstrating *PermLab* I will discuss some of the underlying algorithms, and its design principles. *PermLab* is open source and contributions to the code base from the permutation patterns community are welcomed.

Periodic patterns of k-shifts

Kassie Archer (Dartmouth College)

If $f : A \to A$, where A is a linearly ordered set, we define the pattern of f at x (of length n) as $\mathrm{Pat}(x, f, n) = \rho(x, f(x), f^2(x), \cdots, f^{n-1}(x)) \in S_n$ where ρ is a reduction map that takes in a list of different elements and returns a permutation π by labeling the smallest element in the list with a 1, the second smallest with a 2, and so on. For example, $\rho(3, 6, 2, 3.4, 100, -2) = 352461$.

For instance, if we define the *binary shift* Σ_2 on the set of infinite binary words with lexicographic order by

$$\Sigma_2(w_1w_2w_3\cdots) = w_2w_3w_4\cdots,$$

we have that $\mathrm{Pat}(01101001\cdots, \Sigma_2, 5) = 25413$, since

$$01001\cdots < 01101001\cdots < 1001\cdots < 101001\cdots < 1101001\cdots.$$

The above definition assumes that the values $x, f(x), \dots, f^{n-1}(x)$ are all different. Another interesting case is when $x \in A$ is a n-periodic point of f, that is, $f^n(x) = x$ but $f^i(x) \neq x$ for $1 \leq i < n$. In this case, we say that $[\pi]$ is the *periodic pattern* of f at x if $\mathrm{Pat}(x, f, n) = \pi$. The n-periodic points of the binary shift are exactly those sequences $w = (w_1w_2\cdots w_n)^\infty$ where the word $w_1w_2\cdots w_n$ is primitive (that is, there is no $k > 1$ so that $w_1w_2\cdots w_n = v^k$). For example, the periodic pattern of Σ_2 at $(00101)^\infty$ is [13524].

It was shown in [1] that for any given piecewise monotone function $f : I \to I$, where $I \in \mathbb{R}$ is a closed interval, there exist patterns π that are not realized by f, that is, there is no $x \in I$ with $\mathrm{Pat}(x, f, n) = \pi$.

In [2], Elizalde characterized and enumerated the patterns realized by the k-shift Σ_k (i.e., the shift on k-ary words), which is equivalent to the map $x \mapsto kx \bmod 1$ on the unit interval.

In this talk, we describe and enumerate the *periodic* patterns of a few maps, including the k-shift map and the tent map, which is defined on $[0, 1]$ as

$$x \mapsto \begin{cases} 2x & 0 \leq x \leq 1/2 \\ 2 - 2x & 1/2 < x \leq 1. \end{cases}$$

As a byproduct of the enumeration of periodic patterns of Σ_k, we derive a recursive formula describing the number of cyclic permutations of length n with $k - 1$ descents:

$$C(n, k) = L_k(n) - \sum_{i=2}^{k-1} \binom{n+k-i}{k-i} C(n, i),$$

where

$$L_k(n) = \frac{1}{n} \sum_{d|n} \mu(d) k^{\frac{n}{d}}$$

is the number of length n primitive words on k letters. The number of length n periodic patterns of the k-shift is then $\sum_{i=2}^{n} C(n,i)$.

We also study periodic patterns of the reverse k-shift, which is equivalent to the map $x \mapsto 1 - kx \bmod 1$ on the unit interval. For this purpose, we define an order on infinite sequences on k letters that plays the role that the lexicographic order has for the k-shift. We conjecture that the number of length n periodic patterns of the reverse k-shift is the same as the number of length n periodic patterns of the k-shift, but its relationship to the number of cyclic permutations of length n with $k-1$ ascents is somewhat more complicated in the cases when n is two times an odd number.

This is joint work with Sergi Elizalde.

[1] C. Bandt, G. Keller and B. Pompe, Entropy of interval maps via permutations, *Nonlinearity* 15 (2002), 1595–1602.

[2] S. Elizalde, The number of permutations realized by a shift, *SIAM J. Discrete Math.* 23 (2009), 765–786.

Priority Queues and Pattern Classes

Mike Atkinson (University of Otago)

A priority queue is a container into which new items can be inserted and items removed: the item removed is always the smallest item in the container. A sequence of insert and remove operations therefore transforms an input sequence into an output sequence. Let X be any pattern class and consider the set X* of possible output permutations when the permutations of X are presented as input to a priority queue. In general X* is larger than X but it is still a pattern class. We determine the pattern classes X* that arise when X is taken to have a basis of permutations of length 3. In particular we prove that X* is finitely based in this case. We also give an example of a pattern class X with a basis permutation of length 4 for which X* is not finitely based.

Shape-Wilf-equivalences for vincular patterns

Andrew M. Baxter (Penn State University)

We extend the notion of shape-Wilf-equivalence to vincular patterns (also known as "generalized patterns" or "dashed patterns"), and explore the implications for Wilf-classificiation of the set of vincular patterns. Shape-Wilf-equivalence is a stronger relation than Wilf-equivalence which can lead to families of Wilf-equivalences. In particular we strengthen a result of Elizalde and Kitaev by showing $\sigma \oplus 1$ is shape-Wilf-equivalent to $\tau \oplus 1$ whenever σ and τ are Wilf-equivalent consecutive patterns. We also prove that 1-23 is shape- Wilf-equivalent to 3-12 and that 1-32 is shape-Wilf-equivalent to 3-21. This settles the Wilf-equivalence of 12-3-4, 12-4-3, 21-3-4, and 21-4-3 conjectured by the author and Pudwell, as well as the Wilf-equivalence of 3-12-4, 1-23-4, 1-32-4, and 3-21-4.

Surprising Symmetries in Objects Counted by the Catalan numbers

Miklós Bóna (University of Florida)

Let $S_{n,r}(q)$ be the total number of copies of the pattern q in all r-avoiding permutations of length n. In this paper, we first prove the identities

$$S_{n,132}(312) = S_{n,132}(231) = S_{n,132}(213).$$

The first equality is trivial, but the second one is not. In fact, the two statistics in the second equality are *not* equdistributed, but they have the same cumulative value. A proof using generating functions is relatively straightforward, but we will also present a combinatorial proof. This is the first 3-fold symmetry in Catalan-like objects we have encountered.

Then we significantly generalize our results by presenting a large class of non-trivial equivalences in the above sense for patterns of arbitrary length. The proofs of these generalizations are combinatorial.

Enumeration of permutations sorted with two passes through a stack and D_8 symmetries

Mathilde Bouvel (LaBRI, CNRS, Univ. Bordeaux)

We denote by $\mathbf{S}$ the stack sorting operator on permutations, and by D_8 the eight element group generated by the usual transforms $\mathbf{r}$ (reverse), $\mathbf{c}$ (complement) and $\mathbf{i}$ (inverse). We study the set of permutations that are sorted by $\mathbf{S} \circ \alpha \circ \mathbf{S}$ (denoted $\mathrm{Id}(\mathbf{S} \circ \alpha \circ \mathbf{S})$) for $\alpha \in D_8$. We provide a characterization by (generalized) excluded patterns and enumeration results, that are refined according to a number of usual statistics on permutations.

Theorem 0.1. *The sets of permutations that are sorted by $S \circ \alpha \circ S$, for any α in D_8 are:*

(i) $\mathrm{Id}(\mathbf{S} \circ \mathbf{S}) = \mathrm{Id}(\mathbf{S} \circ \mathbf{i} \circ \mathbf{c} \circ \mathbf{r} \circ \mathbf{S}) = Av(2341, 3\bar{5}241)$;

(ii) $\mathrm{Id}(\mathbf{S} \circ \mathbf{c} \circ \mathbf{S}) = \mathrm{Id}(\mathbf{S} \circ \mathbf{i} \circ \mathbf{r} \circ \mathbf{S}) = Av(231)$;

(iii) $\mathrm{Id}(\mathbf{S} \circ \mathbf{r} \circ \mathbf{S}) = \mathrm{Id}(\mathbf{S} \circ \mathbf{i} \circ \mathbf{c} \circ \mathbf{S}) = Av(1342, 31\text{-}4\text{-}2) = Av(1342, 3\bar{5}142)$;

(iv) $\mathrm{Id}(\mathbf{S} \circ \mathbf{i} \circ \mathbf{S}) = \mathrm{Id}(\mathbf{S} \circ \mathbf{r} \circ \mathbf{c} \circ \mathbf{S}) = Av(3412, 3\text{-}4\text{-}21)$.

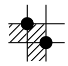
As we know since the seminal work of Knuth, the set $Av(231)$ (of one-stack sortable permutations) is enumerated by the Catalan numbers $Cat_n = \frac{1}{n+1}\binom{2n}{n}$. West has conjectured the set $Av(2341, 3\bar{5}241)$ of two-stack sortable permutations is enumerated by $\frac{2(3n)!}{(n+1)!(2n+1)!}$, and this formula has been proved by Dulucq, Gire and Guibert. For the two other sets, conjectures on their enumeration (refined with the distribution of some statistics) have been proposed by Claesson, Dukes and Steingrímsson. We prove these conjectures, that are stated as Theorems 0.2 and 0.3 below.

Theorem 0.2. *The two sets* $\mathrm{Id}(\mathbf{S}\circ\mathbf{S})$ *and* $\mathrm{Id}(\mathbf{S}\circ\mathbf{r}\circ\mathbf{S})$ *are enumerated according to the size of the permutations by the same sequence. Moreover, the tuple of statistics* (updownword, rmax, lmax, zeil, indmax, slmax, slmax $\circ\mathbf{r}$) *has the same distribution on both sets.*

The updownword statistics associates a word $w \in \{u, d\}^{n-1}$ to each permutation σ of size n, with $w_i = u$ (resp. d) if $\sigma(i) < \sigma(i+1)$ (resp. $\sigma(i) > \sigma(i+1)$). The equidistribution of the statistics updownword implies that the following statistics are also equidistributed in $\mathrm{Id}(\mathbf{S}\circ\mathbf{S})$ and $\mathrm{Id}(\mathbf{S}\circ\mathbf{r}\circ\mathbf{S})$: (des, maj, maj $\circ\mathbf{r}$, maj $\circ\mathbf{c}$, maj $\circ\mathbf{rc}$, valley, peak, ddes, dasc, rir, rdr, lir, ldr). Consequently, the bijection of Theorem 0.2 preserves the *joint* distribution of a 20-tuple of statistics on permutations.

Theorem 0.3. *The set* $\mathrm{Id}(\mathbf{S} \circ \mathbf{i} \circ \mathbf{S})$ *is enumerated by the Baxter numbers*

$$Bax_n = \frac{2}{n(n+1)^2} \sum_{k=1}^{n} \binom{n+1}{k-1}\binom{n+1}{k}\binom{n+1}{k+1}.$$

Moreover, the triple of statistics (des, lmax, comp) *has the same distribution on* $\mathrm{Id}(\mathbf{S} \circ \mathbf{i} \circ \mathbf{S})$ *and on the set* $Av(2\text{-}41\text{-}3, 3\text{-}14\text{-}2)$ *of Baxter permutations. It also has the same distribution than the* *triple of statistics* (lmax, occ_μ, comp) *on the set* Av(2-41-3, 3-41-2) *of twisted Baxter permutations, where* occ_μ *denoted the number of occurrences of the mesh pattern* $\mu =$  .

Theorems 0.2 and 0.3 are proved using generating trees and rewriting systems. Furthermore, the proof of Theorem 0.3 makes use of a recent effective bijection of Giraudo between Baxter permutations, twisted Baxter permutations and pairs of twin binary trees.

This is joint work with Olivier Guibert.

A Fast Algorithm for Permutation Pattern Matching Based on Alternating Runs

Marie-Louise Bruner (Vienna University of Technology)

The PERMUTATION PATTERN MATCHING (PPM) problem asks whether a permutation P can be matched into a permutation T, i.e. whether T conatins P as pattern. It is known that PPM is in general NP-complete. However, when restrictions are made on the input instance efficient algorithms are known. For instance in the case that the pattern P is a separable permutation, PPM can be solved in polynomial time. In this talk I present the first algorithm that improves upon the $O^*(2^n)$ runtime required by brute-force search without imposing restrictions on P and T. The algorithm exploits the decomposition of permutations into alternating runs and has an exponential worst-case runtime of $O^*(1.79^{run(T)})$, where $run(T)$ denotes the number of alternating runs of T. It thus performs particularly well when the involved permutations have few alternating runs.

This talk is based on joint work with Martin Lackner.

A combinatorial proof of joint equidistribution of certain pairs of permutation statistics

Alexander Burstein (Howard University)

We give a direct combinatorial proof of the joint equidistribution of two pairs of permutation statistics, $(\mathsf{aid}, \mathrm{des})$ and $(\mathsf{inv}, \mathsf{lec})$, which have been previously shown to have the same joint distribution as $(\mathrm{maj}, \mathsf{exc})$, the pair of the major index and the number of excendances of a permutation. Moreover, the triple $(\mathsf{inv}, \mathsf{lec}, \mathsf{pix})$ was shown to have the same distribution as $(\mathrm{maj}, \mathsf{exc}, \mathsf{fix})$, where fix is the number of fixed points of a permutation. We define a new statistic aix so that our bijection maps $(\mathsf{inv}, \mathsf{lec}, \mathsf{pix})$ to $(\mathsf{aid}, \mathrm{des}, \mathsf{aix})$.

Consecutive Patterns in up-down permutations

Adrian Duane (Unversity of California, San Diego)

Let A_n denote the set of up-down permutations of length n. For any sequence of distinct integers $\sigma_1, \ldots, \sigma_n$, we define $\mathrm{red}(\sigma)$ to be the permutation that results by replacing the i-th smallest integer in σ by i. If $\tau \in A_{2j}$, then we say that an up-down permutation $\sigma = \sigma_1 \ldots \sigma_n \in A_n$ has a τ-match at position i if $\mathrm{red}(\sigma_i, \sigma_{i+1}, \ldots, \sigma_{i+2j-1}) = \tau$ and we define $\tau\text{-mch}(\sigma)$ to be the number of τ-matches in σ. We say that $\tau \in A_{2j}$ has the *alternating minimal overlapping property* if two τ-matches in an alternating permutation $\sigma \in A_n$ can share at most two letters. For such a τ, we say that $\sigma \in A_{m(2j-2)+2}$ is an *maximal packing* for τ if $\tau\text{-mch}(\sigma) = m$, i.e., σ has the maximum number of possible τ-matches.

Let τ be an up-down permutation of length $2j$ with the alternating minimal overlapping property. We define the *generalized maximum packing polynomial of* τ $GMP_{\tau,2n}(x)$ as follows. Let $\mathcal{L}$ be the set of compositions $\alpha = (2a_1, 2a_2, 2a_3, \ldots, 2a_\ell)$ of $2n$ such that $a_1 \geq 0$, $a_i > 0$ for all $1 < i \leq \ell$, and $a_i = 1 \bmod j-1$ for all even i. Suppose that $\alpha = (2a_1, 2a_2, 2a_3, \ldots, 2a_\ell) \in \mathcal{L}$. Then we let $gmp_\tau(\alpha)$ be the number of permutations $\sigma = \sigma_1 \ldots \sigma_{2n}$ such that if we decompose σ into sequences as $\sigma = \sigma^{(1)} \ldots \sigma^{(\ell)}$ where $\sigma^{(i)}$ has length $2a_i$ for $i = 1, \ldots, \ell$, then (i) $\sigma^{(i)}$ is an increasing sequence if i is odd, (ii) $\mathrm{red}(\sigma^{(i)})$ is a maximum packing for τ if i is even, and (iii) the last element of $\sigma^{(i)}$ is less than the first element of $\sigma^{(i+1)}$ for $i = 1, \ldots, \ell-1$. We define the weight of the composition α to be $wt(\alpha) = gmp_\tau(\alpha)(-1)^{a_1\chi(a_1>0)}(-1)^{\sum_{s\geq 2}(a_{2s}-1)}(x-1)^{\sum_{s\geq 1}\frac{2a_{2s}-2}{2j-2}}$ where for any statement A, $\chi(A) = 1$ if A is true and $\chi(A) = 0$ if A is false. We define $GMP_{\tau,2n}(x) = \sum_{\alpha\in\mathcal{L}} wt(\alpha)$.

Duane and Remmel proved that for any $\tau \in A_{2j}$ with the alternating minimal overlapping property,

$$1 + \sum_{n\geq 1} \frac{t^n}{n!} \sum_{\sigma\in A_{2n}} x^{\tau-mch(\sigma)} = \frac{1}{1 - \sum_{n\geq 1} \frac{t^{2n}}{(2n)!} GMP_{\tau,2n}(x)}.$$

Thus in order to be able to explicitly calculate this generating function, we need to be able to compute $GMP_{\tau,2n}(x)$. In this paper, we focus on the problem of computing $GMP_{\tau,2n}(x)$. We will describe several infinite families of up-down permutations τ with the alternating minimally overlapping property for which $GMP_{\tau,2n}(x)$ can be computed via simple recursions. In such situations, we can compute the generating function $1 + \sum_{n\geq 1} \frac{t^n}{n!} \sum_{\sigma\in A_{2n}} x^{\tau-mch(\sigma)}$.

This is joint work with Jeffrey Remmel.

Parallelogram polyominoes, the sandpile model on $K_{m,n}$, and a q,t-Narayana polynomial

Mark Dukes (University of Strathclyde)

In this talk I will highlight some results from a recent paper which that was motived by a correspondence between bivincular patterns and composition matrices.

We classify recurrent configurations of the sandpile model on the graph $K_{m,n}$ in terms of polyominoes. A canonical toppling process on these recurrent states gives rise to a "bounce" path within the corresponding polyomino. This bounce path gives rise to a polynomial that we call the q,t-Narayana polynomial. We discuss this q,t-Narayana polynomial and its relation to the well-known q,t-Catalan polynomial.

(This is joint work with Yvan Le Borgne.)

Adin-Roichman-Mansour type identities

Sen-Peng Eu (National University of Kaohsiung, and Air Force Academy)

In [1], Adin and Roichman proved analytically the following identities, where $\mathsf{ldes}(\pi)$ denotes the position of the last descent. At the same time, Mansour [2] found a variation for $\mathfrak{S}_n(132)$.

Theorem 0.4 (Adin-Roichman)**.** *Let* $\mathfrak{S}_n(321)$ *be the set of* 321*-avoiding permutations in* $\mathfrak{S}_n$*. The following identities hold.*

$$\sum_{\pi\in\mathfrak{S}_{2n+1}(321)} (-1)^{\mathsf{inv}(\pi)} q^{\mathsf{ldes}(\pi)} = \sum_{\pi\in\mathfrak{S}_n(321)} q^{2\cdot\mathsf{ldes}(\pi)}, \qquad \textit{for } n \geq 0,$$

$$\sum_{\pi\in\mathfrak{S}_{2n}(321)} (-1)^{\mathit{inv}(\pi)} q^{\mathit{ldes}(\pi)} = (1-q) \sum_{\pi\in\mathfrak{S}_n(321)} q^{2\cdot \mathit{ldes}(\pi)}, \qquad \text{for } n \geq 1.$$

Exhausting computer research shows that this "$2n$ reduces to n" phenomenon is indeed rare. In this work, we would like to give several new A-R-M type identities, e.g:

Theorem 0.5. *Let $\mathfrak{B}_n(321)$ be the set of 321-avoiding Baxter permutations in $\mathfrak{S}_n$. For $n \geq 0$, we have*

$$\sum_{\pi\in\mathfrak{B}_{2n+1}(321)} (-1)^{\mathrm{maj}(\pi)} p^{\mathit{fix}(\pi)} q^{\mathrm{des}(\pi)} = p \cdot \sum_{\pi\in\mathfrak{B}_n(321)} p^{2\cdot \mathit{fix}(\pi)} q^{2\cdot\mathrm{des}(\pi)}.$$

Theorem 0.6. *Let $\mathit{Alt}_n(321)$ be the set of 321-avoiding alternating permutations in $\mathfrak{S}_n$, and let $\mathit{lead}(\pi) = \pi_1$, the first entry of π. For all $n \geq 1$, we have*

(i) $$\sum_{\pi\in \mathit{Alt}_{4n+2}(321)} (-1)^{\mathit{inv}(\pi)} \cdot q^{\mathit{lead}(\pi)} = (-1)^{n+1} \sum_{\pi\in \mathit{Alt}_{2n}(321)} q^{2\cdot \mathit{lead}(\pi)}$$

(ii) $$\sum_{\pi\in \mathit{Alt}_{4n+1}(321)} (-1)^{\mathit{inv}(\pi)} \cdot q^{\mathit{lead}(\pi)} = (-1)^{n} \sum_{\pi\in \mathit{Alt}_{2n}(321)} q^{2\cdot \mathit{lead}(\pi)}$$

(iii) $$\sum_{\pi\in \mathit{Alt}_{4n}(321)} (-1)^{\mathit{inv}(\pi)} \cdot q^{\mathit{lead}(\pi)} = (-1)^{n+1}(1-q) \sum_{\pi\in \mathit{Alt}_{2n}(321)} q^{2(\mathit{lead}(\pi)-1)}$$

(iv) $$\sum_{\pi\in \mathit{Alt}_{4n-1}(321)} (-1)^{\mathit{inv}(\pi)} \cdot q^{\mathit{lead}(\pi)} = (-1)^{n}(1-q) \sum_{\pi\in \mathit{Alt}_{2n}(321)} q^{2(\mathit{lead}(\pi)-1)}.$$

Theorem 0.7. *Let $\mathcal{DS}_n(312)$ be the set of 312-avoiding double simsum permutations in $\mathfrak{S}_n$, then*

(i) $$\sum_{\pi\in\mathcal{DS}_{2n+2}(312)} (-1)^{\mathrm{maj}(\pi)} \cdot q^{\mathit{fix}(\pi)} = (-1+q^2) \sum_{\pi\in\mathcal{DS}_n(312)} q^{2\mathit{fix}(\pi)}, \text{ for } n \geq 1.$$

(ii) $$\sum_{\pi\in\mathcal{DS}_{2n-1}(312)} (-1)^{\mathrm{maj}(\pi)} \cdot q^{\mathit{lead}(\pi)} = \frac{2}{q(1+q^2)} \sum_{\pi\in\mathcal{DS}_n(312)} q^{2\mathit{lead}(\pi)}, \text{ for } n \geq 2.$$

These results are co-worked with T.S Fu, Y.J. Pan and P.L. Yan.

[1] R.M. Adin, Y. Roichman, Equidistribution and sign-balance on 321-avoiding permutations, Sémin. Loth. Combin. 51 (2004) B51d. ArXiv:math.CO/0304429.

[2] T. Mansour, Equidistribution and sign-balance on 132-avoiding permutations, Séminaire Lotharingien de Combinatoire 51 (2004) B51e.

The Möbius function of the consecutive pattern poset

Luca Ferrari (University of Firenze)

For the poset of classical permutation patterns, the first results about its Möbius function were obtained by Sagan and Vatter. Further results have been found by Steingrímsson and Tenner and by Burstein, Jelínek, Jelínková and Steingrímsson. The general problem in this case of classical patterns seems quite hard. In contrast, the poset of consecutive pattern containment has a much simpler structure. Here we compute the Möbius function of that poset. In most cases our results give an immediate answer. In the remaining cases, we give a polynomial time recursive algorithm to compute the Möbius function. In particular, we show that the Möbius function only takes the values -1, 0 and 1.

An interesting result to note in connection to this is Björner's one on the Möbius function of factor order. Although that poset is quite different from ours, there are interesting similarities. In particular, both deal with consecutive subwords and the possible values of the Möbius function are -1, 0 and 1 in both cases.

Denote with $\mathcal{P}$ the poset of permutations with respect to consecutive pattern containment, and take $\sigma, \tau \in \mathcal{P}$ such that $\sigma \leq \tau$. In order to present our results, we need a couple of definitions.

Suppose σ occurs in $\tau = a_1 a_2 \dots a_n$. If a_{i+1} is the leftmost letter of τ involved in any occurrence of σ in τ, we say that τ has a *left tail of length i with respect to σ*. Analogously, τ has a *right tail of length j with respect to σ* if a_{n-j} is the rightmost letter of τ involved in any occurrence of σ in τ. If it is clear from the context what σ is, we simply talk about left and right tails of τ.

For example, with respect to the pattern 123, the permutation 286134759 has a left tail of length 3, and a right tails of length 2, since all occurrences of 123 belong within the segment 1347.

The following definition is borrowed from the theory of codes.

Given a permutation τ, its *prefix* (resp. *suffix*) *pattern of length k* is the permutation of length k order isomorphic to the prefix (resp. suffix) of τ of length k. In other words, the prefix (resp. suffix) pattern of length k of τ is the unique permutation $\sigma \in S_k$ such that τ has a left (resp. right) tail of length 0 with respect to σ. In case the prefix and suffix patterns of length k of τ coincide, we say that τ has a *bifix pattern of length k*.

In the case where σ occurs precisely once in τ, we show that $\mu(\sigma, \tau)$ depends only on the lengths, a and b, of the two tails of τ with respect to σ. More precisely, $\mu(\sigma, \tau)$ is 1 if $a = b \leq 1$, it is -1 if $a = 0$ and $b = 1$ or vice versa, and 0 otherwise (in which case τ has a tail of length at least 2).

Our main result deals with intervals $[\sigma, \tau]$ where σ occurs at least twice in τ. This result implies that, as in the case of one occurrence, if τ has a tail of length at least 2, then $\mu(\sigma, \tau) = 0$. In the remaining cases, where the tails of τ have length at most 1, the main result gives a recursive algorithm for computing $\mu(\sigma, \tau)$, by producing, if possible, an element $\mathcal{C}$ in $[\sigma, \tau]$, where $|\mathcal{C}| < |\tau| - 2$, such that $\mu(\sigma, \tau) = \mu(\sigma, \mathcal{C})$. This element $\mathcal{C}$, if it exists, must be a bifix pattern of τ, and it must lie below the two elements covered by τ, but not below the element obtained by deleting one letter from each end of τ. If no such element $\mathcal{C}$ exists (which is most often the case), we have $\mu(\sigma, \tau) = 0$.

(This is joint work with Antonio Bernini and Einar Steingrimsson.)

Bounds for the number of permutations containing a low density of patterns

Benjamin Fineman (University of California, Davis)

We seek to find a result similar to the Stanley-Wilf conjecture, but for permutations containing a low density of a certain pattern. In our case, instead of an exponential bound, we find a bound that is exponentially suppressed, and show that such a bound is indeed necessary.

Previous work in pattern avoidance has focused on finding bounds for the number of permutations with no occurrences of a given pattern. Let $S_n(\gamma)$ be the number of permutations in S_n that avoid the pattern γ. For any pattern of length three, $S_n(\gamma) = C_n$, the n-th Catalan number. In general, it is not the case that $S_n(\gamma)$ only depends of the length of γ. Bóna showed that $S_n(1234) < S_n(1324)$, and these are the only other patterns with known formulae. The most sweeping result concerning bounds for the number of pattern avoiding permutations is the Stanley-Wilf conjecture, recently proved by Marcus and Tardos.

Theorem 1 (Stanley-Wilf conjecture, 1980)**.** *Let γ be any pattern. Then there exists a constant c so that for all positive integers, we have $S_n(\gamma) \leq c^n$.*

Let γ be a pattern of length f, and let $\chi_\delta^n(\gamma)$ be the number of permutations in S_n with fewer than $\delta^f n^f$ f-patterns. Our goal is to prove the following theorem:

Theorem 2. *For every f, $\delta < 1/(2f)$, there are N, a, b, such that for $n > N$, we have*

$$(a^n)n! \leq \chi_\delta^n(\gamma) \leq (b^n)n!$$

In particular, we have $a = \delta^f/2$, and $b = \left(\frac{e}{(f-1)\delta}\right)^\delta \left(\frac{f-1}{f}\right)^{1/f} + t$ for any $t > 0$.

Note that the bound we have is indeed non trivial. The term $\left(\frac{e}{(f-1)\delta}\right)^{\delta}$ approaches 1, as $\delta \to 0$, and the second term $\left(\frac{f-1}{f}\right)^{1/f}$ is constant smaller than 1, depending only on the length of the permutation γ. Furthermore, we can choose t small enough, so that the quantity

$$\left(\frac{e}{(f-1)\delta}\right)^{\delta}\left(\frac{f-1}{f}\right)^{1/f}+t$$

is strictly smaller than 1, giving a nontrivial bound. In particular, for a pattern of length 3, the theorem above, with choice of $\delta = .001$ and $t = .01$ implies that for n sufficiently large, there are at most

$$n!\left\{\left(\frac{e}{(2).001}\right)^{.001}\left(\frac{2}{3}\right)^{1/3}+.01\right\}^{n} < n!(.9)^{n}$$

patterns with fewer than $.001n^3$ 132-patterns.

Covering all n-permutations with $(n+1)$-permutations

Anant Godbole (East Tennessee State University)

Let Σ_n be the set of all permutations on $[n] := \{1, 2, \ldots, n\}$. We denote by κ_n the smallest cardinality of a subset $\mathcal{A}$ of Σ_{n+1} that "covers" Σ_n, in the sense that each $\pi \in \Sigma_n$ may be found as an order-isomorphic subsequence of some π' in $\mathcal{A}$. What are general upper bounds on κ_n? If we randomly select ν_n elements of Σ_{n+1}, when does the probability that they cover Σ_n transition from 0 to 1? Can we provide a fine-magnification analysis that provides the "probability of coverage" when ν_n is around the level given by the phase transition? In this talk we answer these questions and raise others. This is joint work with Bill Kay (USC, Columbia), Taylor Allison (NC State), and Katie Hawley (Harvey Mudd) – and partially answers a question raised by Robert Brignall at last year's PP Conference.

Pattern Avoidance in Ordered Partitions

Adam M. Goyt (Minnesota State University Moorhead)

Pattern avoidance in permutations, words, and set partitions have all been studied individually and in conjunction with one another. In this talk we will meld the concepts of pattern avoidance in set partitions with pattern avoidance in permutations in a slightly different way by considering pattern avoidance in ordered set partitions. A *partition* of $[n] = \{1, 2, \ldots, n\}$ is a family of nonempty disjoint sets $B_1, B_2, \ldots, B_k$ called *blocks*, that satisfy $\bigcup_{i=1}^{k} B_i = [n]$. In a set partition, we list the blocks in order of increasing minimal elements and we list the elements in each block in increasing order. In an *ordered set partition* we keep the increasing order on the elements within a block and impose order on the blocks. For example, 36/27/1/45 is an ordered set partition, and 27/45/36/1 is a different ordered set partition, despite the fact that the underlying set partition is the same.

We will say that an ordered partition $\sigma = B_1/B_2/\ldots/B_k$ of $[n]$ contains a copy of a permutation $p = p_1p_2\ldots p_m \in S_m$ if there is a sequence of elements $a_{i_1}a_{i_2}\ldots a_{i_m}$ such that $a_{i_j} \in B_{i_j}$ for $1 \le j \le m$, $i_1 < i_2 < \cdots < i_m$, and $a_{i_1}a_{i_2}\ldots a_{i_m}$ is order isomorphic to p. We will give enumerative results for sets of ordered partitions which avoid a permutation pattern of length 3. We will also discuss how ordered partitions are related to words, and give a simple bijection showing that the number of words avoiding 123 is the same as the number of words avoiding 132.

This is joint work with Anant P. Godbole, Jennifer Herdan, and Lara K. Pudwell.

Random permutations (and beyond)

Jennie Hansen (Heriot-Watt University)

In this talk we view uniform random permutations as part of a continuum of random mapping models and we investigate the component structure of the random mappings in this continuum as the mappings become (in some sense) more like permutations. Specifically, let $[n] = \{1, 2, ..., n\}$, let $\mathcal{M}_n$ denote the set of all mappings $f : [n] \to [n]$, and let $S_n \subset \mathcal{M}_n$ denote the set of all permutations $\sigma : [n] \to [n]$. Any mapping $f \in \mathcal{M}_n$ can be represented by a directed graph $G(f)$ on vertices labelled $1, 2, ..., n$ where there is a directed edge $i \to j$ in $G(f)$ if and only if $f(i) = j$. So if $\sigma \in S_n$, then $G(\sigma)$ is the directed graph that represents the cycle structure of σ and every vertex in $G(f)$ has in-degree 1. More generally, if $f \in \mathcal{M}_n$, then the connected components of $G(f)$ consists of directed cycles with directed trees attached to the cycles and vertices can have in-degree greater than 1. If T is a random element of $\mathcal{M}_n$, then $G(T)$ is a random directed graph and we can investigate random variables that are determined by the structure of the digraph $G(T)$. One such random variable is $C_1(T)$, the size of the component in $G(T)$ which contains the vertex labelled 1 (i.e. the size of a 'typical' component). It is well-known that is σ_n is a random permutation on $[n]$, then $C_1(\sigma_n)$ is uniformly distributed on $[n]$. In this talk we consider the exact and asymptotic distributions of $C_1(T_{n,a})$ where, for $0 \leq a \leq n$, $T_{n,a}$ is a random element of $\mathcal{M}_n$ such that the vertices in the digraph $G(T_{n,a})$ have at least $n - a$ vertices with in-degree 1 and at most a vertices with in-degree 2 and such that $T_{n,0} = \sigma_n$. (We note that, in some sense, the smaller the value of a relative to n, the 'closer' the random mapping $T_{n,a}$ is to the random permutation σ_n). The results obtained in this talk are based on urn scheme arguments and use a calculus developed by the authors for random mappings with exchangeable in-degrees.

This is joint work with Jerzy Jaworski (Adam Mickiewicz University), who was supported by the Marie Curie Intra-European Fellowship No. 236845 (RANDOMAPP) within the 7th European Community Framework Programme.

PatternClass: A GAP Package for Permutation Pattern Classes

Ruth Hoffmann (University of St Andrews)

Many interesting pattern classes of permutations give rise to regular languages using the rank encoding [AAR03]. This includes the classes generated by finite Token Passing Networks (TPNs) as well as others. A remarkable result of [AAR03] is that in this situation the basis of the class also has a regular language of encodings, and that the basis can be computed from the language representing the whole class, and vice versa.

This talk will present the PatternClass GAP [GAP08] package, which includes: building pattern classes from TPNs with and without a token constraint [ARL04]; computing the basis of a class and vice versa; computing the class from the minimal avoidance set and the token constraint; and inspecting whether a given class can be simulated by a TPN. Furthermore, there are methods for rank encoding and decoding as well as some statistical inspections, including calculating the spectrum of a class [ALR05], and printing the list of permutations of a specific length that are contained within a class.

The talk will also discuss ongoing developments with the package including: computation of plus- and minus-indecomposable sub-languages [AA05]; checking if a given permutation is simple [Bri08] or if it belongs to a given class; calculating the direct and skew sum of classes [AAV10]; and implement one point deletion in simple permutations. Consequently from some of these ideas it should be possible to compute chains of simple permutations [AD12] and separable classes [AAV10] amongst other features.

We will demonstrate the package and its workings, as well as show insight to the ideas behind the algorithms used.

Expected Patterns in Permutations Avoiding 123

Cheyne Homberger (University of Florida)

In the set of all patterns in S_n, it is clear that each k-pattern occurs equally often. If we instead restrict to the class of permutations avoiding a specific pattern, the situation quickly becomes more interesting. Miklós Bóna recently proved that, surprisingly, if we consider the class of permutations avoiding the pattern 132, all other non-monotone patterns of length 3 are equally common. In this talk I examine the class $Av(123)$, and give exact formula for the occurrences of each length 3 pattern. While this class does not break down as nicely as $Av(132)$, we find some interesting similarities between the two and prove that the number of 231 patterns is the same in each.

Stanley-Wilf limits of layered patterns

Vít Jelínek (Charles University)

We prove that the Stanley-Wilf limit of any layered permutation of length k is at most $4k^2$, which is tight up to a multiplicative costant. For specific layered patterns, we are able to give more precise upper bounds: notably, we prove that the Stanley-Wilf limit of the pattern 1324 is at most 16.

These bounds follow from a general result showing that any permutation avoiding a pattern of a special form can be obtained by merging two permutations, each of which avoids a smaller pattern.

(This is joint work with Anders Claesson and Einar Steingrimsson)

Generating functions for permutations with no consecutive pattern matches within the cycles

Miles Jones (University of California, San Diego)

Given a sequence $\sigma = \sigma_1 \dots \sigma_n$ of distinct integers, let $\text{red}(\sigma)$ be the permutation found by replacing the i^{th} largest integer that appears in σ by i. For example, if $\sigma = 2\ 7\ 5\ 4$, then $\text{red}(\sigma) = 1\ 4\ 3\ 2$. Let Υ be a set of permutations and let σ be a permutation in S_n with k cycles $C_1 \dots C_k$. Then we say that σ has a *cycle* Υ*-match* (c-Υ-match) if there exists an i such that $C_i = (c_{0,i}, \dots, c_{p_i-1,i})$ and an r such that $\text{red}(c_{r,i} \dots c_{r+j-1,i}) \in \Upsilon$ where we take indices of the form $r+s$ modulo p_i. Let $\mathcal{NCM}_n(\Upsilon)$ be the set of all permutations $\sigma \in S_n$ such that σ has no cycle Υ-match. We have been able to get closed form generating functions of the following form for certain sets of patterns Υ.

$$NCM_\Upsilon(t) = \sum_{n\geq 0} \frac{t^n}{n!} |\mathcal{NCM}_n(\Upsilon)|$$

Results: Let Γ be the set of all permutations $\sigma = \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5 \in S_5$ such that

$$\sigma_1 < \sigma_2 > \sigma_3 < \sigma_4 > \sigma_5.$$

Let $\Upsilon_1 = \Gamma \cup \{1234\}$ then

$$NCM_{\Upsilon_1}(t) = \frac{2e^{t^2/2}e^{t^4/12}}{2-2t+t^2e^{-t}}.$$

Let $\Upsilon_2 = \Gamma \cup \{132, 1234\} = \{132, 1234, 35241, 45231, 34251\}$ then

$$NCM_{\Upsilon_2}(t) = \frac{2e^t e^{t^2/2}}{4 - 2e^t + t^2 + 2t}.$$

Let $\Upsilon_3 = \Gamma \cup \{231, 1234\} = \{231, 1234, 13254, 14253, 15243\}$ then

$$NCM_{\Upsilon_3}(t) = \frac{e^t e^{t^2/2}}{-1 - t + 2e^t - te^t}.$$

On permutation boxed mesh patterns

Sergey Kitaev (University of Strathclyde)

Mesh patterns are a generalization of vincular patterns. Mesh patterns were introduced by Branden and Claesson to provide explicit expansions for certain permutation statistics as, possibly infinite, linear combinations of (classical) permutation patterns.

We introduce the notion of a boxed mesh pattern and study avoidance of these patterns on permutations. We prove that the celebrated former Stanley-Wilf conjecture is not true for all but eleven boxed mesh patterns; for seven out of the eleven patterns the former conjecture is true, while we do not know the answer for the remaining four (length-four) patterns. Moreover, we show that an analogue of a well-known theorem of Erdos and Szekeres does not hold for boxed mesh patterns of lengths larger than 2. Finally, we discuss enumeration of permutations avoiding simultaneously two or more length-three boxed mesh patterns, where we meet generalized Catalan numbers.

This is joint work with Sergey Avgustinovich and Alexander Valyuzhenich.

Random Superpatterns

Martha Liendo (East Tennessee State University)

The number of *preferential arrangements* or *rankings* of length a on an alphabet of size a are given by the so-called ordered Bell numbers $B(a) = \sum_{k=1}^{a} k!S(a,k)$, where $S(a,k)$ are the Stirling numbers of the second kind. A word of length n that contains all preferential arrangements of length a is called a *superpattern.* It is known by joint work of Burstein, Hästö, and Mansour that the minimum length $n(a,a)$ of a superpattern satisfies $n(a,a) \le a^2 - 2a + 4$ and it conjectured that $n(a,a) = a^2 - 2a + 4$. In this talk we will focus on alphabets of size 2 and 3 and consider a sequence $X_1, X_2, \ldots$ of independent and identically distributed variables, each taking the value j with probability $1/a; a = 2, 3$. The distribution of the waiting time W till the sequence becomes a superpattern is obtained in closed form, as are the generating function and moments. For example, it is shown for $a = 3$ that

$$p(n) = P(W = n) = \frac{6}{3^n} \sum_{m=7}^{n} [(n-4)^2 - 2] \binom{n-2}{m-2}.$$

This is joint work with Anant Godbole.

Permuted Basement Fillings, k-ary Trees, and Watermelons

Janine LoBue (University of California, San Diego)

Symmetric functions are important objects of study which illustrate the connection between algebra, representation theory, and combinatorics. In particular, the Schur functions are a notable basis for the symmetric functions because they have a combinatorial interpretation as the generating function of column-strict tableaux, as well as representation-theoretic value as the irreducible characters of the symmetric group. A q-t-analogue of the Schur functions are the symmetric Macdonald polynomials, introduced by Macdonald in 1988, from which the Schur functions can be obtained by setting q=t=0. Even more general are the nonsymmetric Macdonald polynomials, of which the symmetric Macdonald polynomials are a special case. In 2007, Haglund, Haiman, and Loehr gave a combinatorial interpretation to these nonsymmetric Macdonald polynomials, namely fillings of certain diagrams with positive integer entries. Since then, Mason has studied the polynomials $\widehat{E}_\gamma$ that result from setting q=t=0 in these nonsymmetric Macdonald polynomials. These can be considered a nonsymmetric refinement of the Schur functions, and are generated by fillings of certain diagrams, indexed by weak compositions, with basement permutation equal to the identity. When this basement is permuted to equal σ, we obtain the combinatorial objects of interest, known as *permuted basement fillings*. These fillings generate the polynomials $\widehat{E}_\gamma^\sigma$, which decompose the Schur functions.

Since the Schur functions are known to possess many nice properties, there is much interest in which elements of that structure are maintained by the the $\widehat{E}_\gamma^\sigma$s. Much of the study of PBFs has focused on cataloging the algebraic properties common to the Schur functions and $\widehat{E}_\gamma^\sigma$s, but not much work has been done on enumerating the permuted basement fillings, or PBFs, which generate the $\widehat{E}_\gamma^\sigma$s. One prominent question is whether there is an analogue of the hook formula for PBFs. Unfortunately, since the requirements for being a PBF are much stricter than the requirements for being a tableau, and some of these requirements are quite complicated, there does not seem to be anything analogous to the hook formula in this general situation. When we fix a simple enough shape, however, we can count the number of PBFs of that shape.

In this talk, I will discuss how one can count the number of PBFs of certain basic shapes, including all rectangular shapes. We will find that these objects, which have come to be a topic of study primarily because of their algebraic significance, also have connections to familiar combinatorial objects including k-ary trees, lattice paths, and watermelons. Aside from enumerating these permuted basement fillings, we will begin to look at certain statistics to find q-analogues of these results. For example, for a certain class of PBFs counted by k-ary trees, we will give a bijection to lattice paths and see how a descent between entries in the top row of a PBF corresponds to a certain behavior in the path. Further study of patterns and statistics within these PBFs seems likely to yield interesting results, as there is much yet to be discovered about these objects.

Stack-sorting and preimages of mesh patterns

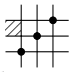
Hjalti Magnússon (Reykjavik University)

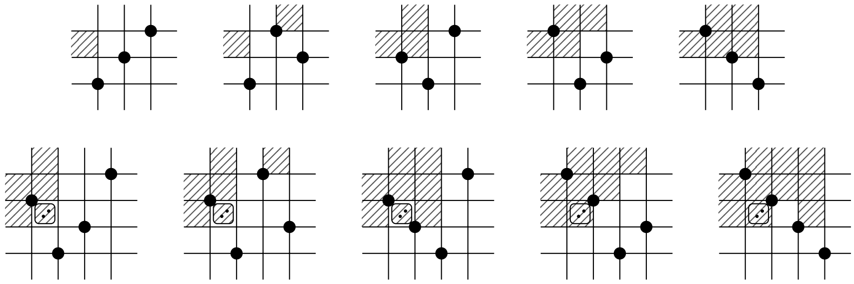
In the 1960s, Knuth showed that permutations avoiding the pattern 231 are the permutations sortable with a single pass through a stack. In 1993, West [4] classified permutations sortable with two passes through a stack, using *barred patterns*. Using *mesh patterns*, introduced by Brändén and Claesson in [1], Claesson and Úlfarsson [2] implemented an algorithm which automates West's proof.

More precisely, given a classical pattern p and the stack-sort operator S, the algorithm generates a set of mesh patterns M, such that for each permutation π, π avoids all $m \in M$ if and only if $S(\pi)$ avoids p. Thus, by taking $p = 231$, West's results can be reproduced.

The patterns output by the algorithm are no longer classical, and thus we cannot apply the algorithm again to obtain a description of permutations sortable by three passes through a stack. Recently, however, Úlfarsson, in [3], defined *decorated patterns*, and used them to give a description of permutations sortable with three passes through a stack.

We extend the original algorithm, of Úlfarsson and Claesson, to handle mesh patterns with a single shaded box. This allows us to fully automate Úlfarsson's proof.

For example, given the pattern $p =$ , the algorithm gives us that the permutations that avoid p after one pass through a stack are exactly the permutations that avoid the following 10 patterns:



This is joint work with Henning Úlfarsson.

Generalized Interval Embeddings

Brian Miceli (Trinity University)

Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ and let $\mathbb{N}^*$ denote the set of all words over $\mathbb{N}$. Let ϵ denote the empty word. Given words u and v in $\mathbb{N}^*$, we say that u is a factor of v if there are words w_1 and w_2 such that $v = w_1 u w_2$. In such a situation, we say u is a suffix of v if $w_2 = \epsilon$. Given $u = u_1 u_2 \ldots u_\ell \in \mathbb{N}^*$, we define the *norm of* u to be $\Sigma u = u_1 + u_2 + \cdots + u_\ell$ and we define the *length of* u to be $|u| = \ell$. We then allow x and t to be commuting variables and we define the *weight of* u to be $wt(u) = x^{\Sigma u} t^{|u|}$.

Given any poset $\mathcal{P} = (\mathbb{N}, \leq_p)$ and $m, n \in \mathbb{N}$, we let $I^{\mathcal{P}}_{m,\infty} = \{k \in \mathbb{N} : m \leq_p k\}$ and $I^{\mathcal{P}}_{m,n} = \{n \in \mathbb{N} : m \leq_p k \leq_p n\}$. Given any words $u = u_1 \ldots u_k$ and $w = w_1 w_2 \ldots w_\ell$ in $\mathbb{N}^*$, we say that u *embeds into* w *relative to* $\mathcal{P}$, written $u \leq_{\mathcal{P}} w$ if there is a factor $w' = w'_1 w'_2 \ldots w'_k$ of w such that $w'_i \in I^{\mathcal{P}}_{u_i,\infty}$ for every $1 \leq i \leq k$. We define $S^{\mathcal{P}}(u)$ to be the set of all words w that embed u such that the only embedding of u into w occurs at the right end of w, and we set

$$\mathcal{S}^{\mathcal{P}}(u, x, t) = \sum_{w \in S^{\mathcal{P}}(u)} wt(w).$$

Given $u, v \in \mathbb{N}^*$, u and v are $\mathcal{P}$-*Wilf equivalent*, written as $u \sim_{\mathcal{P}} v$, if $\mathcal{S}^{\mathcal{P}}(u, x, t) = \mathcal{S}^{\mathcal{P}}(v, x, t)$. Kiteav, Liese, Remmel and Sagan [1] studied various properties of $\mathcal{P}$-Wilf Equivalence where $\mathcal{P}$ is the standard order on $\mathbb{N}$ and Langley, Liese, and Remmel [2] studied various properties of $\mathcal{P}_k$-Wilf equivalence where $\mathcal{P}_k = (\mathbb{N}, \leq_k)$ and $i \leq_k j$ if and only if $i \equiv j \mod k$ and $i < j$.

We study a generalization $\mathcal{P}$-Wilf equivalence based on intervals. That is, suppose that we are given a poset $\mathcal{P} = (\mathbb{N}, \leq_p)$ and a sequence $\vec{U}$ of intervals $(\{I^{\mathcal{P}}_{m_1,n_1}, I^{\mathcal{P}}_{m_2,n_2}, \ldots, I^{\mathcal{P}}_{m_k,n_k}\}$ where either $m_i \leq_p n_i$ and $m_i, n_i \in \mathbb{N}$ or $m_i \in \mathbb{N}$ and $n_i = \infty$. Then we say that w has an *interval-embedding of* $\vec{U}$ *into* w *relative to* $\mathcal{P}$, denoted $\vec{U} \leq_{\mathcal{P}} w$ if there is a factor $w' = w'_1 w'_2 \ldots w'_k$ of w such that $w'_i \in I^{\mathcal{P}}_{m_i,n_i}$ for every $1 \leq i \leq \ell$. We then define $S^{\mathcal{P}}(\vec{U})$ to be the set of all words $w = w_1 \ldots w_n \in \mathbb{N}^*$ such that $n \geq k$, there is an interval embedding of $\vec{U}$ into the suffix of w of length k, and there is no interval embedding of $\vec{U}$ into $w_1 \ldots w_{n-1}$. We set

$$\mathcal{S}^{\mathcal{P}}(\vec{U}, x, t) = \sum_{w \in S^{\mathcal{P}}(\vec{U})} wt(w),$$

and given two sequences $\vec{U}$ and $\vec{V}$ of intervals of $\mathcal{P}$, we say that $\vec{U}$ is $\mathcal{P}$-Wilf equivalent to $\vec{V}$, written as $\vec{U} \sim_{\mathcal{P}} \vec{V}$, if $\mathcal{S}^{\mathcal{P}}(\vec{U}, x, t) = \mathcal{S}^{\mathcal{P}}(\vec{V}, x, t)$.

We show that under mild assumptions on $\mathcal{P}$, $S^{\mathcal{P}}(\vec{U})$ is accepted by a finite automaton and, hence, $\mathcal{S}^{\mathcal{P}}(\vec{U}, x, t)$ is a rational function. We compute $\mathcal{S}^{\mathcal{P}}(\vec{U}, x, t)$ for various special cases of $\vec{U}$ and use these computations to establish various non-trivial Wilf-equivalences in this setting.

[1] S. Kitaev, J. Liese, J. Remmel, and B.E. Sagan, Rationality of generalized containments in words and Wilf equivalence, *Electron. J. Combin.*, **16(2)** (2009), R22

[2] T. Langley, J. Liese, and J. Remmel, Generating functions for Wilf equivalence under the generalized factor order, *J. Integer Seq.*, **14** (2011), 11.4.2

This is joint work with Jeffrey Liese and Jeffrey Remmel.

Permutations with exactly r occurrences of a length three pattern

Brian Nakamura (Rutgers University)

We consider the problem of enumerating permutations that contain exactly r occurrences of a pattern. In previous work, Markus Fulmek gave an approach to find such generating functions for length three patterns by translating permutations into generalized Dyck paths where certain jumps are allowed. In particular, Fulmek was able to find the generating functions for the $r = 1$ and $r = 2$ cases of 312 as well as the $r = 1$ and $r = 2$ cases of 321. In this talk, we discuss Fulmek's approach and show some ways how it can be automated and extended for more occurrences.

Two-stacks sorting is polynomial

Adeline Pierrot (LIAFA, Université Paris Diderot)

In this article we give a polynomial algorithm to decide whether a given permutation σ is sortable by 2-stacks in series. Given $\sigma = \sigma_1\sigma_2 \ldots \sigma_n$ and two stacks H and V, at each time step we can take the next element of σ and push it onto H, or pop the topmost element of H and push it onto V or pop the topmost element of V and write it in the output. The question is whether there exist a sequence of operations leading to the identity in the output. This problem arises first in Knuth's book The Art of Computer Programming in 1973. Several subclasses or special cases have been solved, either by restricting the operations, the input permutations or taking special kind of stacks. The problem of deciding whether a given permutation σ is sortable by 2-stacks in series has been conjectured to be both NP-complete and polynomial in different articles or books.

Our polynomial algorithm is based onto a previous article in *Permutation Patterns 2011* where we study 2-stacks pushall sortable permutations, that is permutations such that all elements are first pushed onto the stacks H and V before the first element being output. Using the characterization by a coloring of 2-stacks pushall sortable permutations, we can encode by a graph the possible sortings of a given permutation. Indeed, given the right-to-left minima of the permutation, we compute iteratively the graph, the leftmost right-to-left minima corresponding to the pushall case.

This is joint work with Dominique Rossin.

Non-contiguous pattern avoidance in binary trees

Lara K. Pudwell (Valparaiso University)

In 2010, Rowland considered pattern avoidance in rooted ordered binary trees with the following definition: binary tree T contains binary tree t if and only if T contains t as a contiguous rooted ordered subgraph. In this talk, we modify Rowland's definition such that binary tree T contains tree t if and only if there is a sequence of edge contractions of T that produce tree T* which contains t as a rooted ordered subgraph. While Rowland's tree patterns are analogous to consecutive permutation patterns, this new definition is analogous to classical permutation patterns. We completely classify Wilf-classes of trees avoiding a single non-contiguous binary tree pattern and provide generating functions that enumerate pattern-avoiding trees according to number of leaves. We also consider trees that avoid multiple tree patterns simultaneously and provide bijective relationships between certain sets of pattern-avoiding trees and sets of pattern-avoiding permutations.

This is joint work with Mike Dairyko, Samantha Tyner, and Casey Wynn.

Up-down ascent sequences and the q-Genocchi numbers

Jeffrey Remmel (University of California, San Diego)

The Genocchi number G_{2n} for $n \geq 1$ can be defined through its relation with the Bernoulli numbers $G_{2n} = 2(2^{2n} - 1)B_n$ or through its exponential generating function

$$\frac{2t}{e^t + 1} = t + \sum_{n \geq 1} (-1)^n G_{2n} \frac{t^{2n}}{(2n)!}.$$

The median Genocchi numbers H_{2n+1} are defined by $H_{2n+1} = \sum_{i \geq 0} G_{2n-2i} \binom{n}{2i+1}$. The Genocchi numbers have been given combinatorial interpretations by Dumont [3], Dumont and Viennot [4], Burstein et. al. [2], and others.

Zeng and Zhou [6] defined a q-analogue of the Genocchi numbers by defining a q-analogue of the so-called Seidel triangle for the Genocchi numbers by defining polynomials $(g_{i,j}(q))_{i,j \geq 1}$ by $g_{1,1}(q) = g_{2,1}(q) = 1$ and

$$g_{2i+1,j}(q) = g_{2i+1,j-1}(q) + q^{j-1} g_{2i,j}(q), \text{ for } j = 1, 2, \ldots, i+1, \quad (1)$$

$$g_{2i,j}(q) = g_{2i,j+1}(q) + q^{j-1} g_{2i-1,j}(q), \text{ for } j = i, i-1, \ldots, 1, \quad (2)$$

where $g_{i,j}(q) = 0$ if $j < 0$ or $j > \lceil i/2 \rceil$ by convention. They defined the q-Genocchi number $G_{2n}(q)$ and the median q-Genocchi number $H_{2n-1}(q)$ by

$$G_{2n}(q) = g_{2n-1,n}(q) \text{ and } H_{2n-1}(q) = q^{n-2} g_{2n-1,1}(q).$$

We give a new combinatorial interpretation of the elements of the q-analogue of the Seidel triangle in terms of q-counting a up-down ascent sequences. Ascent sequences were introduced by Bousquet-Mélou, Claesson, Dukes, and Kitaev in [1] to study the problem of enumerating $(\mathbf{2}+\mathbf{2})$-free posets. A sequence $(a_1, \ldots, a_n) \in \mathbb{N}^n$ is an *ascent sequence of length* n if and only if it satisfies $a_1 = 0$ and $a_i \in [0, 1 + \text{asc}(a_1, \ldots, a_{i-1})]$ for all $2 \leq i \leq n$. Here, for any integer sequence $(a_1, \ldots, a_i)$, the number of *ascents* of this sequence is

$$\text{asc}(a_1, \ldots, a_i) = |\{j : a_j < a_{j+1}\}|.$$

For any $n \geq 1$, we let Asc_n denote the set of all ascent sequences of length n. Then we say that $a = a_1 \ldots a_n \in Asc_n$ is an *up-down* ascent sequence if $a_1 < a_2 > a_3 < a_4 > \cdots$. Let $UDA_n^{(i)}$ denote the set of elements $a = a_1 \ldots a_n \in UDA_n$ such that $a_n = i$. If $n \geq 1$ and $a = a_1 \ldots a_n \in UDA_n$, then we define the weight of a, $w(a)$, by

$$w(a) = \sum_{i=1}^{n-1} (a_i - \chi(i \text{ even})) \quad (3)$$

where for any statement A, $\chi(A) = 1$ is A is true and $\chi(A) = 0$ is A is false. We prove the following theorem.

Theorem 3. *For all* $1 \leq j \leq \lceil i/2 \rceil$,

$$g_{2i,j}(q) = \sum_{a=a_1 \ldots a_{2i+1} \in UDA_{2i+1}^{(j-1)}} q^{w(a)} \text{ and}$$
$$g_{2i+1,j}(q) = \sum_{a=a_1 \ldots a_{2i+2} \in UDA_{2i+2}^{(j)}} q^{w(a)}.$$

It follows for all $n \geq 1$,

$$G_{2n}(q) = g_{2n-1,n}(q) = \sum_{\substack{a=a_1 \ldots a_{2n} \in UDA_{2n} \\ a_{2n}=n}} q^{w(a)}. \tag{4}$$

and

$$H_{2n-1}(q) = q^{n-2} g_{2n-1,1}(q) = q^{n-2} \sum_{\substack{a=a_1 \ldots a_{2n} \in UDA_{2n} \\ a_{2n}=1}} q^{w(a)}. \tag{5}$$

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Asymptotics of push-all permutations

Dominique Rossin (LIX, CNRS, École Polytechnique)

A long-standing problem in permutation enumeration is to count the number of permutations which can be sorted using two stacks in series. A subclass of these is presented in [2]: the *push-all-sortable permutations*, those which can be sorted by a procedure in which at a given moment all the elements are found in the stacks at once. In other words, all of the pushes onto the first stack are accomplished before the first pop from the second stack is carried out. In that paper, the authors present a polynomial algorithm for deciding whether a permutation is push-all sortable; it is still unknown whether such a polynomial algorithm exists for the larger class. This algorithm relies on a characterisation of the push-all-sortable permutations in terms of a certain bicolouring of their elements, the two colours corresponding to which of the two stacks an element inhabits after all the elements have been loaded into the stacks. In the present work, we make use of this colouring to find the Wilf constant for the class of push-all-sortable permutations; this is a fortiori a lower bound for the Wilf constant of the larger class of permutations sortable by two stacks. More precisely, we find a bijection between certain admissible colourings, which overcount our permutations in a linear fashion, and rooted ternary trees, which have a well-known enumeration yielding the asymptotic formula $(27/4)^n$. This is the same asymptotic as that corresponding to a different subclass of the permutations sortable on two stacks, West's *two-stack-sortable permutations*.

This is joint work with Adeline Pierrot and Julian West.

[1] D. Knuth, The Art of Computer Programming
[2] A. Pierrot and D. Rossin, On two-stack push-all permutations, PP 2011
[3] R.E. Tarjan, Sorting using networks of queues and stacks

Sorting with modified devices

Rebecca Smith (SUNY Brockport)

Knuth showed that a permutation π can be sorted by a stack (meaning that by applying push and pop operations to the sequence of entries $\pi(1), \ldots, \pi(n)$ we can output the sequence $1, \ldots, n$) if and only if π avoids the permutation 231, i.e., if and only if there do not exist three indices $1 \leq i_1 < i_2 < i_3 \leq n$ such that $\pi(i_1), \pi(i_2), \pi(i_3)$ are in the same relative order as 231.

Many similarly structured devices such as pop-stacks, deques, restricted deques (of which stacks are one type), etc. have also been considered. Some of the more robust machines are very difficult to devise an optimal algorithm for (particularly when more than one machine is acting on the permutation). Because of this, the weaker machines are sometimes more practical to study. We continue this tradition by looking at combinations of some of the traditional restrictions on sorting devices.

Extending from bijections between marked occurrences of patterns to all occurrences of patterns

Mark Tiefenbruck (University of California, San Diego)

Consider two problems presented recently at Permutation Patterns, the first posed by Claesson and Linusson, the second posed by Jones and Remmel.

First, in a permutation σ, we define the pattern p such that an occurrence is a subsequence $\sigma_i\sigma_{i+1}\sigma_j$ where $\sigma_i = \sigma_j + 1$ and $\sigma_i < \sigma_{i+1}$. A matching is a partition of the set $\{1, 2, \ldots, 2n\}$ into pairs (i, j) such that $i < j$. In a matching, the pairs (i, l) and (j, k) form a nesting if $i < j$ and $k < l$. In particular, we define a left-nesting to be a nesting where $j = i + 1$, and we define a right-nesting to be a nesting where $l = k + 1$. Claesson and Linusson conjectured that the number of left-nestings in matchings that have no right-nestings has the same distribution as the number of occurrences of p in the permutations in S_n.

Second, let $w = (w_1w_2 \cdots w_k)$ be a cycle in a permutation. A cycle-match of the pattern π is a subsequence of consecutive elements of the cycle, where w_1 follows w_k, that have the same relative order as the entries in π. Jones and Remmel showed that if π begins with 1, then the number of cycle-matches of π in the cycles of the permutations in S_n has the same distribution as the number of consecutive occurrences of π in the permutations in S_n. They conjectured this was true for any π that cannot cover a cycle with overlapping π-cycle-matches. For example, in the cycle (31425), 3142 and 4253 are 3142-cycle-matches that cover the cycle, whereas no cycle can be covered by 2143-cycle-matches.

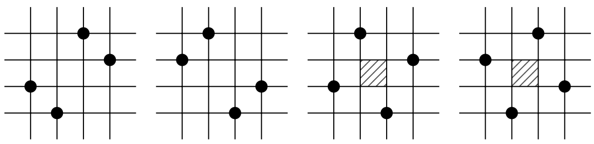
We will present a general technique for showing that two sets of patterns have the same joint distribution. This technique reduces the problem to finding a bijection that preserves a given number of "marked" patterns, which is generally easier. We may augment this technique with the Garsia-Milne involution principle to obtain a bijection that preserves all of the patterns. We will use this technique to solve the above problems and present other interesting results in the study of permutation patterns.

This is joint work with Jeffrey Remmel.

Automated discovery of permutation patterns

Henning Ulfarsson (Reykjavik University)

A substantial amount of research has been devoted to showing that many properties of permutations, as well as objects related to them, are captured by permutation patterns. Examples include properties such as sorting through various devices and smoothness properties of Schubert varieties. Often one needs generalized notions of patterns like barred, vincular and mesh patterns. We have developed an algorithm that takes as input a finite set of permutations and outputs the minimal patterns that the set avoids. Here minimality means that any other pattern avoided by the set of permutations is a consequence of the outputted patterns. This ensures that we get a concise description. The algorithm can for instance discover the description of stack-sortable permutations in terms of avoidance of 231, West-2-stack-sortable permutations in terms of one classical and one barred pattern, forest-like permutations (corresponding to factorial Schubert varieties) in terms of one classical pattern and one vincular pattern. Since the algorithm only takes finite sets of permutations as input, it can never prove that the description it finds is the correct one. One example of a new conjecture the algorithm has generated is that permutations whose Young tableaux (under the RSK-correspondence) are hook-shaped are the permutations avoiding four patterns:



We will discuss the implementation of the algorithm (in Sage (`www.sagemath.org`)).

This is joint work with Anders Claesson.

Lehmer code transforms and Mahonian statistics on permutations

Vincent Vajnovszki (Université de Bourgogne)

In 2000 Babson and Steingrímsson introduced the notion of vincular patterns in permutations. They shown that essentially all well-known Mahonian permutation statistics can be written as combinations of such patterns. Also, they proved and conjectured that other combinations of vincular patterns are still Mahonian. These conjectures were proved later: by Foata and Zeilberger in 2001, and by Foata and Randrianarivony in 2006.

In this paper we give an alternative proof of some of these results. Our approach is based on permutation codes which, like Lehmer's code, map bijectively permutations onto subexcedant sequences. More precisely, we give several code transforms (i.e., bijections between subexcedant sequences) which when applied to Lehmer's code yield new permutation codes which count occurrences of some vincular patterns.

Small permutation classes

Vincent Vatter (University of Florida)

Much of the early work in permutation patterns was motivated by the Stanley-Wilf Conjecture, which stated that every nontrivial permutation class has a finite *(upper) growth rate*,

$$\overline{\mathrm{gr}}(\mathcal{C}) = \limsup_{n\to\infty} \sqrt[n]{|\mathcal{C}_n|},$$

where $\mathcal{C}_n$ denotes the set of permutations of length n in the permutation class $\mathcal{C}$. While Marcus and Tardos elegantly resolved this conjecture (in the affirmative) in 2004, we still know very little about these numbers. In particular, which numbers can occur as growth rates of permutation classes?

At Permutation Patterns 2007, I presented the following result, extending earlier work of Kaiser and Klazar [4].

Theorem 4 (Vatter [5])**.** *Let κ denote the unique positive root of $x^3 - 2x^2 - 1$, approximately* 2.20557. *If the upper growth rate of $\mathcal{C}$ is less than κ then $\mathcal{C}$ has a proper growth rate which is either* 0, 2, *a root of one of the four polynomials*

(P1) $x^3 - x^2 - x - 3$,

(P2) $x^4 - x^3 - x^2 - 2x - 3$,

(P3) $x^4 - x^3 - x^2 - 3x - 1$,

(P4) $x^5 - x^4 - x^3 - 2x^2 - 3x - 1$,

or a root of one of the three families of polynomials

(F1) $x^{k+1} - 2x^k + 1$,

(F2) $(x^3 - 2x^2 - 1)x^{k+\ell} + x^\ell + 1$, *or*

(F3) $(x^3 - 2x^2 - 1)x^k + 1$

for integers $k \geq 1$ and $\ell \geq 0$.

The number κ is the threshold of a sharp phase transition: there are only countably many permutation classes of growth rate less than κ, but uncountably many of growth rate κ. Furthermore, it is the first growth rate at which permutation classes may contain infinite antichains, which in turn is the cause of much more complicated structure. For this reason we single out classes of growth rate less than κ as *small*.

While Theorem 4 characterizes the asymptotics of small permutation classes, it does not give their fine structure, and in particular it says nothing about their exact enumeration. In this talk I will discuss recent joint work with Michael Albert and Nik Ruškuc, in which we were able to complete the structural characterization of small classes, leading to the following result.

Theorem 5 (Albert, Ruškuc, and Vatter [3])**.** *All small permutation classes have rational generating functions.*

The techniques involved in the proof of Theorem 5 involve the substitution decomposition, originally studied by Albert and Atkinson [1], and the geometric grid classes of Albert, Atkinson, Bouvel, Ruškuc, and Vatter [2].

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[4] Tomáš Kaiser and Martin Klazar, *On growth rates of closed permutation classes*, Electron. J. Combin. **9** (2003), no. 2, Research paper 10, 20 pp.

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Bruhat graphs and pattern avoidance

Alexander Woo (University of Idaho)

Associated to each permutation (or, more generally, any element in a Coxeter group) is a graph called the Bruhat graph. We show that the permutations whose Bruhat graphs can be drawn on the plane or on the torus can be characterized by avoiding specific long lists of patterns. My motivation for this question comes from the observation that some properties of Schubert varieties are characterized by avoiding a long list of patterns but none are known so far to require an infinite list of ordinary patterns. Since these properties depend only on the Bruhat graph, the question arises as to whether there is a purely combinatorial explanation for finiteness.

This is joint work with Christopher Conklin with some further contributions from Michael Eldredge.

List of participants

1. Michael Albert
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4. Mike Atkinson
5. Frédérique Bassino
6. Andrew Baxter
7. David Bevan
8. Mathilde Bouvel
9. Marie-Louise Bruner
10. Alexander Burstein
11. Anders Claesson
12. Daniel Daly
13. Adrian Duane
14. Mark Dukes
15. Sen-Peng Eu
16. Luca Ferrari
17. Benjamin Fineman
18. Anant Godbole
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20. Stuart Hannah
21. Jennie Hansen
22. Jennifer Herdan
23. Ruth Hoffmann
24. Cheyne Homberger
25. Sophie Huczynska
26. Vít Jelínek
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28. Bill Kay
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30. Martha Liendo
31. Janine LoBue
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36. Adeline Pierrot
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41. Einar Steingrimsson
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43. Chien-Tai Ting
44. Henning Ulfarsson
45. Vincent Vajnovszki
46. Vince Vatter
47. Alexander Woo