Extending from bijections between marked occurrences of patterns to all occurrences of patterns

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Consider two problems presented recently at Permutation Patterns, the first posed by Claesson and Linusson, the second posed by Jones and Remmel.

First, in a permutation σ, we define the pattern p such that an occurrence is a subsequence $\sigma_i\sigma_{i+1}\sigma_j$ where $\sigma_i = \sigma_j + 1$ and $\sigma_i < \sigma_{i+1}$. A matching is a partition of the set $\{1, 2, \ldots, 2n\}$ into pairs (i, j) such that $i < j$. In a matching, the pairs (i, l) and (j, k) form a nesting if $i < j$ and $k < l$. In particular, we define a left-nesting to be a nesting where $j = i + 1$, and we define a right-nesting to be a nesting where $l = k+1$. Claesson and Linusson conjectured that the number of left-nestings in matchings that have no right-nestings has the same distribution as the number of occurrences of p in the permutations in S_n.

Second, let $w = (w_1w_2 \cdots w_k)$ be a cycle in a permutation. A cycle-match of the pattern π is a subsequence of consecutive elements of the cycle, where w_1 follows w_k, that have the same relative order as the entries in π. Jones and Remmel showed that if π begins with 1, then the number of cycle-matches of π in the cycles of the permutations in S_n has the same distribution as the number of consecutive occurrences of π in the permutations in S_n. They conjectured this was true for any π that cannot cover a cycle with overlapping π-cycle-matches. For example, in the cycle (31425), 3142 and 4253 are 3142-cycle-matches that cover the cycle, whereas no cycle can be covered by 2143-cycle-matches.

We will present a general technique for showing that two sets of patterns have the same joint distribution. This technique reduces the problem to finding a bijection that preserves a given number of "marked" patterns, which is generally easier. We may augment this technique with the Garsia-Milne involution principle to obtain a bijection that preserves all of the patterns. We will use this technique to solve the above problems and present other interesting results in the study of permutation patterns.

This is joint work with Jeffrey Remmel.