Asymptotics of push-all permutations

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A long-standing problem in permutation enumeration is to count the number of permutations which can be sorted using two stacks in series. A subclass of these is presented in [2]: the *push-all-sortable permutations*, those which can be sorted by a procedure in which at a given moment all the elements are found in the stacks at once. In other words, all of the pushes onto the first stack are accomplished before the first pop from the second stack is carried out. In that paper, the authors present a polynomial algorithm for deciding whether a permutation is push-all sortable; it is still unknown whether such a polynomial algorithm exists for the larger class. This algorithm relies on a characterisation of the push-all-sortable permutations in terms of a certain bicolouring of their elements, the two colours corresponding to which of the two stacks an element inhabits after all the elements have been loaded into the stacks. In the present work, we make use of this colouring to find the Wilf constant for the class of push-all-sortable permutations; this is a fortiori a lower bound for the Wilf constant of the larger class of permutations sortable by two stacks. More precisely, we find a bijection between certain admissible colourings, which overcount our permutations in a linear fashion, and rooted ternary trees, which have a well-known enumeration yielding the asymptotic formula $(27/4)^n$. This is the same asymptotic as that corresponding to a different subclass of the permutations sortable on two stacks, West's *two-stack-sortable permutations*.

This is joint work with Adeline Pierrot and Julian West.

[1] D. Knuth, The Art of Computer Programming
[2] A. Pierrot and D. Rossin, On two-stack push-all permutations, PP 2011
[3] R.E. Tarjan, Sorting using networks of queues and stacks