Two-stacks sorting is polynomial

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In this article we give a polynomial algorithm to decide whether a given permutation σ is sortable by 2-stacks in series. Given $\sigma = \sigma_1\sigma_2\dots\sigma_n$ and two stacks H and V, at each time step we can take the next element of σ and push it onto H, or pop the topmost element of H and push it onto V or pop the topmost element of V and write it in the output. The question is whether there exist a sequence of operations leading to the identity in the output. This problem arises first in Knuth's book The Art of Computer Programming in 1973. Several subclasses or special cases have been solved, either by restricting the operations, the input permutations or taking special kind of stacks. The problem of deciding whether a given permutation σ is sortable by 2-stacks in series has been conjectured to be both NP-complete and polynomial in different articles or books.

Our polynomial algorithm is based onto a previous article in *Permutation Patterns 2011* where we study 2-stacks pushall sortable permutations, that is permutations such that all elements are first pushed onto the stacks H and V before the first element being output. Using the characterization by a coloring of 2-stacks pushall sortable permutations, we can encode by a graph the possible sortings of a given permutation. Indeed, given the right-to-left minima of the permutation, we compute iteratively the graph, the leftmost right-to-left minima corresponding to the pushall case.

This is joint work with Dominique Rossin.