Pattern Avoidance in Ordered Partitions

Adam M. Goyt (Minnesota State University Moorhead)

Pattern avoidance in permutations, words, and set partitions have all been studied individually and in conjunction with one another. In this talk we will meld the concepts of pattern avoidance in set partitions with pattern avoidance in permutations in a slightly different way by considering pattern avoidance in ordered set partitions. A *partition* of $[n] = \{1, 2, \ldots, n\}$ is a family of nonempty disjoint sets $B_1, B_2, \ldots, B_k$ called *blocks*, that satisfy $\bigcup_{i=1}^{k} B_i = [n]$. In a set partition, we list the blocks in order of increasing minimal elements and we list the elements in each block in increasing order. In an *ordered set partition* we keep the increasing order on the elements within a block and impose order on the blocks. For example, 36/27/1/45 is an ordered set partition, and 27/45/36/1 is a different ordered set partition, despite the fact that the underlying set partition is the same.

We will say that an ordered partition $\sigma = B_1/B_2/\ldots/B_k$ of $[n]$ contains a copy of a permutation $p = p_1p_2\ldots p_m \in S_m$ if there is a sequence of elements $a_{i_1}a_{i_2}\ldots a_{i_m}$ such that $a_{i_j} \in B_{i_j}$ for $1 \leq j \leq m$, $i_1 < i_2 < \cdots < i_m$, and $a_{i_1}a_{i_2}\ldots a_{i_m}$ is order isomorphic to p. We will give enumerative results for sets of ordered partitions which avoid a permutation pattern of length 3. We will also discuss how ordered partitions are related to words, and give a simple bijection showing that the number of words avoiding 123 is the same as the number of words avoiding 132.

This is joint work with Anant P. Godbole, Jennifer Herdan, and Lara K. Pudwell.