Submodular maximization over easy knapsack constraints

Akshay Gupte

akshay.gupte@ed.ac.uk

School of Mathematics, University of Edinburgh

A submodular function over the $\{0, 1\}$ lattice is regarded as a discrete analog of a convex function, and appears commonly in many combinatorial optimization problems. It can be minimised in polynomial-time, but maximisation is NP-hard although it is 1/2-approximable. Many approximation algorithms are known for maximising over different classes of independence families, such as constant number of matroids and $\leq$-knapsacks. We consider the question over the intersection of an independence family with a collection of $\leq$- and $\geq$-knapsacks that satisfy a certain property that is related to integrality of their covering polytope and their clutters. The feasible points in these knapsacks can be obtained by monomial orderings of binary vectors. We show that when k (number of knapsacks) is bounded by a constant then the maximum can be approximated to the same factor as that for submodular maximisation over the independence family. We also give a lower bound on approximability by establishing that there does not exist a randomised algorithm with approximation factor roughly $\Omega(\sqrt{k}/2^{\log n})$. This is established by showing reducibility of a large class of cardinality maximization problems to a combinatorial question that we propose for ordering integers under different permutations.