Laguerre digraphs and continued fractions

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(This talk is based on joint work with Alexander Dyachenko, Matthias Pétréolle, Alan Sokal.)

A *Laguerre digraph* is a directed graph on n vertices such that each vertex has indegree 0 or 1 and outdegree 0 or 1. Thus, the connected components are either directed cycles or directed paths. They are a combinatorial interpretation of the coefficients of the Laguerre polynomials. A Laguerre digraph with no directed paths and only directed cycles is simply the digraph of a permutation in cycle notation. We will begin by introducing Laguerre digraphs.

We then introduce Flajolet's combinatorial theory of continued fractions and state a continued fraction identity for the series $\sum_{n=0}^{\infty} n!$ due to Euler (1760). There are several proofs known for this identity; our work focuses on two bijective proofs due to Foata–Zeilberger (1990) and Biane (1993).

In a series of recent papers with Dyachenko, Pétréolle and Sokal, we have analysed the intermediate steps in the Foata–Zeilberger and Biane bijections. The "Biane history" involves building up a Laguerre digraph by inserting vertices at each stage, whereas the "Foata–Zeilberger history" involves building up a Laguerre digraph by inserting edges at each stage. We will show simple examples to illustrate both histories without going into any technical details.

In [1], we solved conjectured continued fractions due to Randrianarivony–Zeng (1996), Sokal–Zeng (2022), and Deb–Sokal (arxiv:2022) using the Foata–Zeilberger history. In [3, 2], we extended various permutation statistics to Laguerre digraphs and used them along with the Biane bijection to combinatorially interpret the Stieltjes–Rogers matrices of Sokal–Zeng's second continued fraction for permutations. This approach also partly helps in solving a conjecture of Corteel–Sokal (2017) on the Hankel total-positivity of the Laguerre polynomials. We will end the talk by quickly stating some of these results.

No prerequisites will be required for this talk.

[1] B. Deb, Continued fractions using a Laguerre digraph interpretation of the Foata–Zeilberger bijection and its variants, in preparation.

[2] B. Deb, A. Dyachenko, M. Pétréolle and A.D. Sokal, Lattice paths and branched continued fractions III: Generalizations of the Laguerre, rook and Lah polynomials, in preparation.

[3] B. Deb, A.D. Sokal, Continued fractions for cycle-alternating permutations, arXiv preprint: https://arxiv.org/abs/2304.06545