# Well-quasi-ordering permutations

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(This talk is based on joint work with Vincent Vatter.)

The study of well-quasi-ordering in combinatorics includes some of the most celebrated results of the past 70 years, including Higman's Theorem, Kruskal's Tree Theorem and Robertson & Seymour's Graph Minor Theorem. The general set-up is to consider a family of combinatorial structures (such as graphs or permutations), and some form of ordering on this family (typically an embedding of smaller structures into larger ones, such as graph minor, induced subgraph, or permutation containment). Such an ordered family is *well-quasi-ordered* (wqo) if it contains no infinite antichains – that is, an infinite set of structures no two of which are comparable in the ordering.

The combinatorial structure of choice for most of this talk is the permutation, equipped with containment (which is the natural 'induced substructure' order). We know of an abundance of different infinite antichains of permutations (so the set of all permutations is certainly not wqo), but mostly researchers are interested in sets (or *classes*) which comprise the permutations that *avoid* some given set. Here we can ask: is a given class wqo?

This talk will survey recent developments in wqo for permutations, and the relationship with other notions of interest (such as the enumeration of permutation classes). A class being wqo is often seen as an indicator that the class is 'tame' (whereas those with infinite antichains are 'wild'), although a recent result (involving uncountably many different wqo classes of permutations) suggests that wqo classes are not as tame as we might have hoped. On the other hand, a stronger notion, known as *labelled* well-quasi-ordering, perhaps offers a better guarantee of 'tameness'.