Permutations that separate close elements

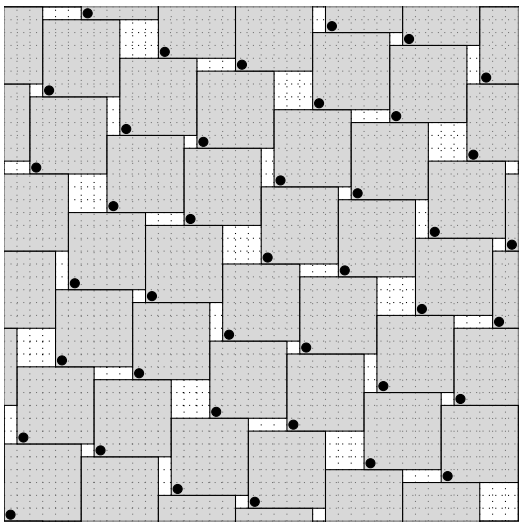
Simon R. Blackburn

s.blackburn@rhul.ac.uk

Royal Holloway University of London

(This talk is based on joint work with Tuvi Etzion.)

Look at the diagram below. It is drawn on an $n \times n$ torus (here with $n = 40$). There are n non-overlapping 6×6 squares; more generally, we will consider rectangles that are s cells wide and k cells high. The dots in the lower left-hand corners form a permutation: there is one dot in each row and each column. For fixed n and k, what is the largest value $\sigma(k, n)$ of s where such a construction is possible?



I proved (*J.Combin. Theory Ser. A*, 2023) that $\sigma(n, k)$ can only take one of two values: $\sigma(n, k) \in \{\lfloor (n-1)/k \rfloor - 1, \lfloor (n-1)/k \rfloor\}$. This establishes a conjecture of Mammoliti and Simpson from 2020. Tuvi Etzion and I have recently shown which of these two values $\sigma(n, k)$ takes, determining the value of $\sigma(n, k)$ for all values of n and k. In this talk, I will discuss these results and some of the techniques we use.