Sorting with modified devices

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Knuth showed that a permutation π can be sorted by a stack (meaning that by applying push and pop operations to the sequence of entries $\pi(1), \ldots, \pi(n)$ we can output the sequence $1, \ldots, n$) if and only if π avoids the permutation 231, i.e., if and only if there do not exist three indices $1 \leq i_1 < i_2 < i_3 \leq n$ such that $\pi(i_1), \pi(i_2), \pi(i_3)$ are in the same relative order as 231.

Many similarly structured devices such as pop-stacks, deques, restricted deques (of which stacks are one type), etc. have also been considered. Some of the more robust machines are very difficult to devise an optimal algorithm for (particularly when more than one machine is acting on the permutation). Because of this, the weaker machines are sometimes more practical to study. We continue this tradition by looking at combinations of some of the traditional restrictions on sorting devices.