Stack-sorting and preimages of mesh patterns

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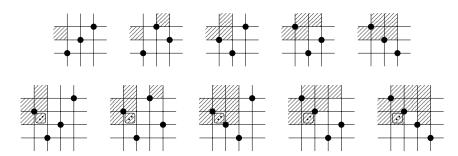
In the 1960s, Knuth showed that permutations avoiding the pattern 231 are the permutations sortable with a single pass through a stack. In 1993, West [4] classified permutations sortable with two passes through a stack, using *barred patterns*. Using *mesh patterns*, introduced by Brändén and Claesson in [1], Claesson and Úlfarsson [2] implemented an algorithm which automates West's proof.

More precisely, given a classical pattern p and the stack-sort operator S, the algorithm generates a set of mesh patterns M, such that for each permutation π , π avoids all $m \in M$ if and only if $S(\pi)$ avoids p. Thus, by taking p = 231, West's results can be reproduced.

The patterns output by the algorithm are no longer classical, and thus we cannot apply the algorithm again to obtain a description of permutations sortable by three passes through a stack. Recently, however, Úlfarsson, in [3], defined *decorated patterns*, and used them to give a description of permutations sortable with three passes through a stack.

We extend the original algorithm, of Úlfarsson and Claesson, to handle mesh patterns with a single shaded box. This allows us to fully automate Úlfarsson's proof.

For example, given the pattern $p = \frac{1}{2}$, the algorithm gives us that the permutations that avoid p after one pass through a stack are exactly the permutations that avoid the following 10 patterns:



This is joint work with Henning Úlfarsson.