A combinatorial proof of joint equidistribution of certain pairs of permutation statistics

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We give a direct combinatorial proof of the joint equidistribution of two pairs of permutation statistics, $(\mathsf{aid}, \mathrm{des})$ and $(\mathsf{inv}, \mathsf{lec})$, which have been previously shown to have the same joint distribution as $(\mathrm{maj}, \mathsf{exc})$, the pair of the major index and the number of excendances of a permutation. Moreover, the triple $(\mathsf{inv}, \mathsf{lec}, \mathsf{pix})$ was shown to have the same distribution as $(\mathrm{maj}, \mathsf{exc}, \mathsf{fix})$, where fix is the number of fixed points of a permutation. We define a new statistic aix so that our bijection maps $(\mathsf{inv}, \mathsf{lec}, \mathsf{pix})$ to $(\mathsf{aid}, \mathrm{des}, \mathsf{aix})$.