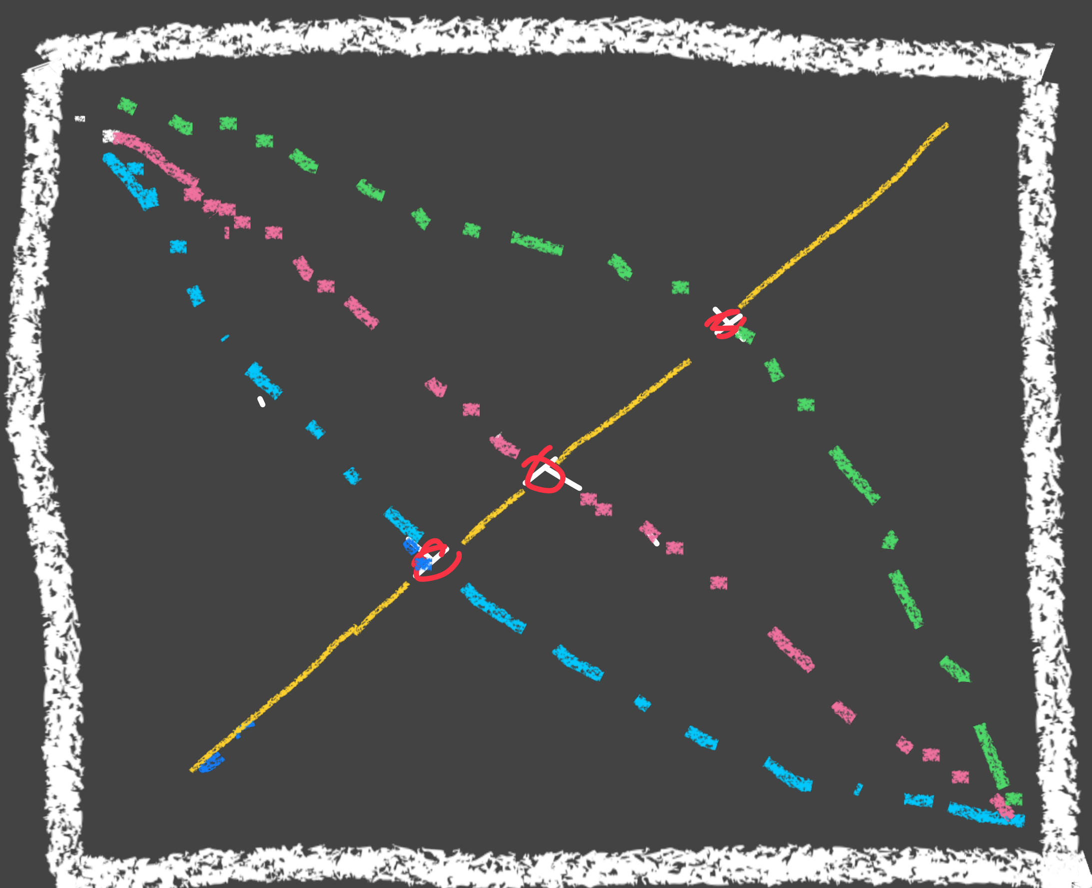
# Fixed points of permutations avoiding an increasing sequence



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joint w/
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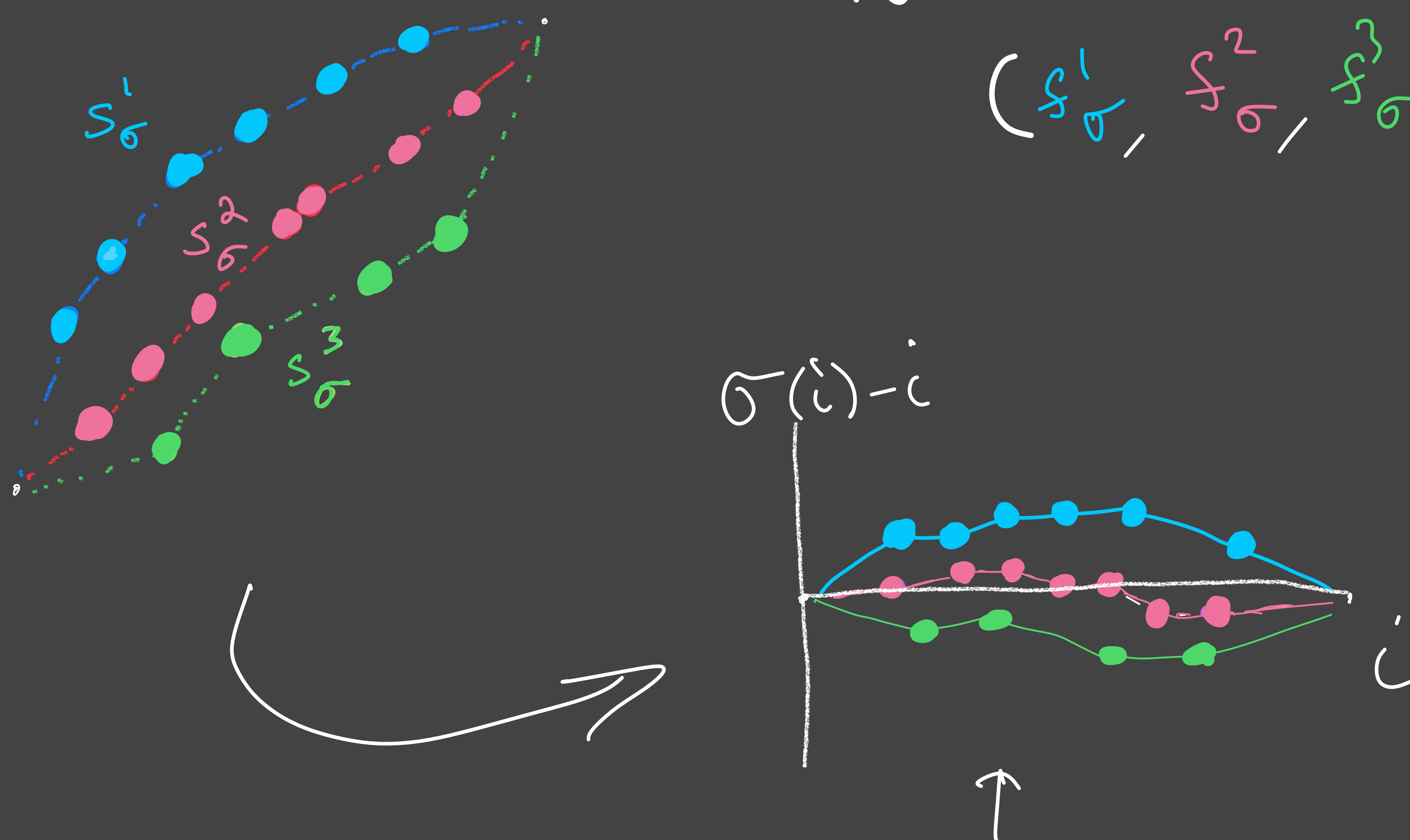
From $\sigma \in Av(4321)$ to points $(s_\sigma^1, s_\sigma^2, s_\sigma^3)$

to functions $(f_\sigma^1, f_\sigma^2, f_\sigma^3)$



- $\Lambda = (\lambda_1, \dots, \lambda_d)$ is a collection of Brownian bridges conditioned to sum to zero & not intersect.
  the traceless Dyson Brownian bridge.

- $P_\sigma(t) = \frac{1}{\sqrt{2dn}} \left( f_\sigma^1(\lceil nt \rceil), \dots, f_\sigma^d(\lceil nt \rceil) \right)$

**Thm** (Hoffman, Rizzolo, S. '20)

$$P_\sigma \xrightarrow{\text{dist}} \Lambda$$

- $M(\sigma) = \sum_{i=1}^{n} \mathbb{1}_{\sigma(i)=i} \, \delta_{(\sigma(i)-i)/\sqrt{2dn}}$

**Thm** (Hoffman, Rizzolo, S. 2021)

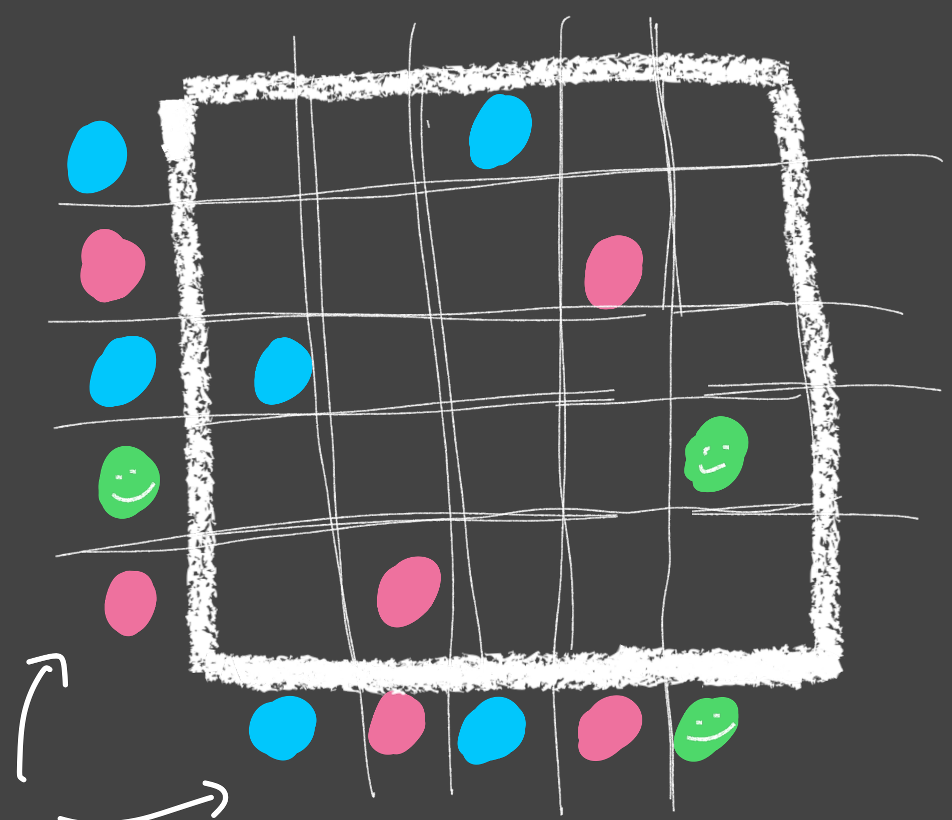
Let $\sigma \in \mathrm{Av}_n(12\cdots d(d+1))$ be uniformly random, let $\Lambda = (\lambda_1, \dots, \lambda_d)$ be a traceless Dyson Brownian bridge, and $\{X_\ell\}_{\ell=1}^{d}$ independent Bernoulli$(\frac{1}{2d})$ r.v.s.

$$M(\sigma) \xrightarrow[\text{dist}]{} \sum_{\ell=1}^{d} X_\ell \, \delta_{\lambda_\ell(1/2)}$$

**Cor.** (Hoffman, Rizollo, S. '21)

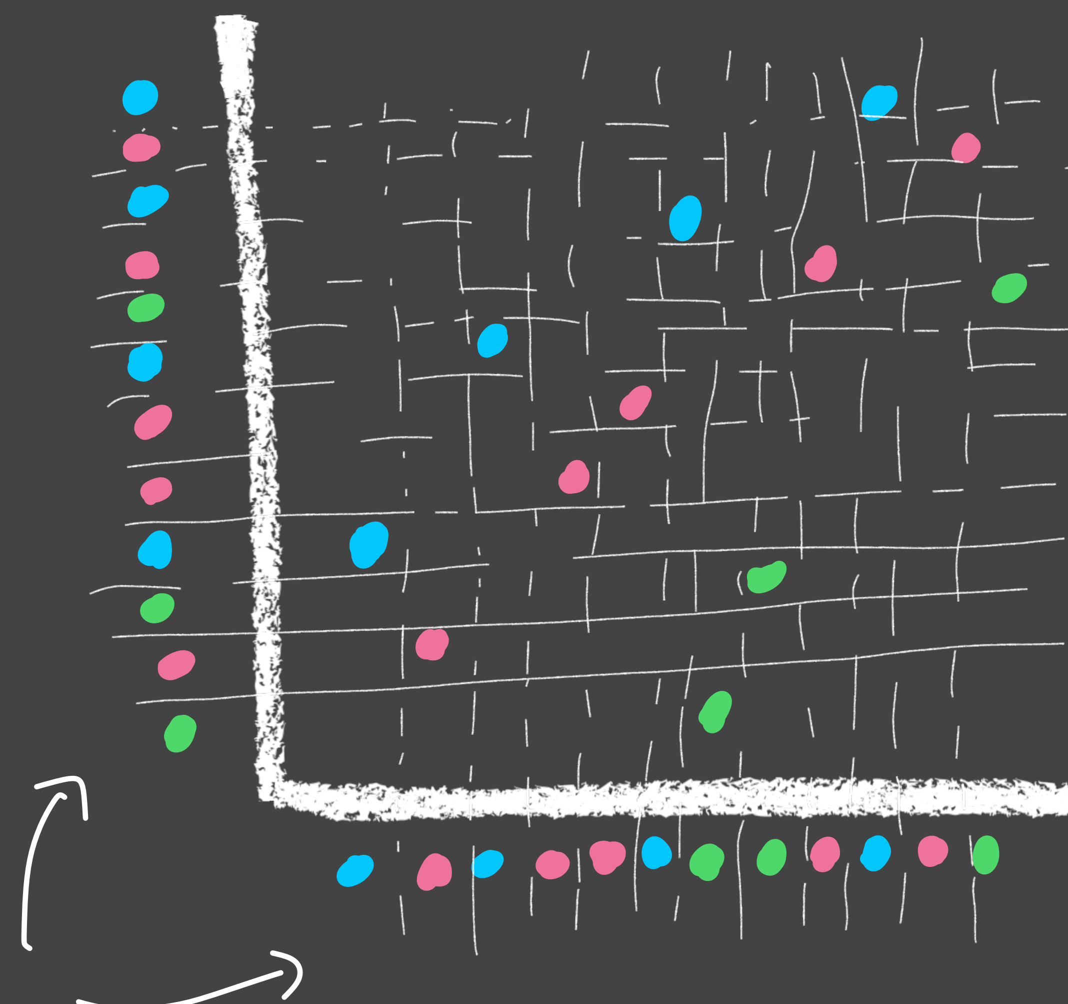
The no. of fixed points of a random permutation uniform in $Av_n(12\cdots d(d+1))$ are distributed like the sum of $d$ independent Bernoulli$\left(\frac{1}{2d}\right)$ r.v.s

Projection of $Av_n((d+1)d \cdots 21)$ into $[d]^n \times [d]^n$



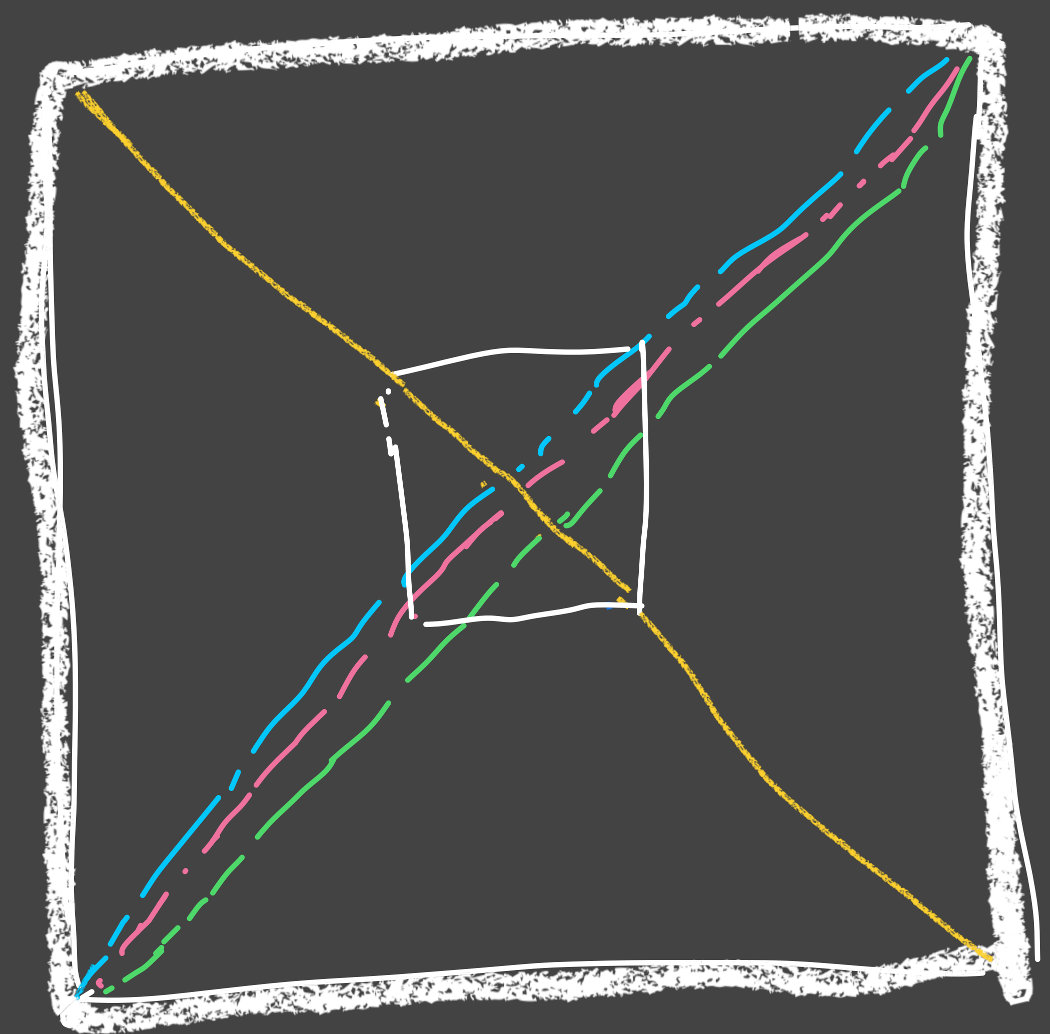
Match colors in increasing order to create permutation

Projection of $\Delta_{v_n}((d+1)d \ldots 21)$ to $[d]^n \times [d]^n$

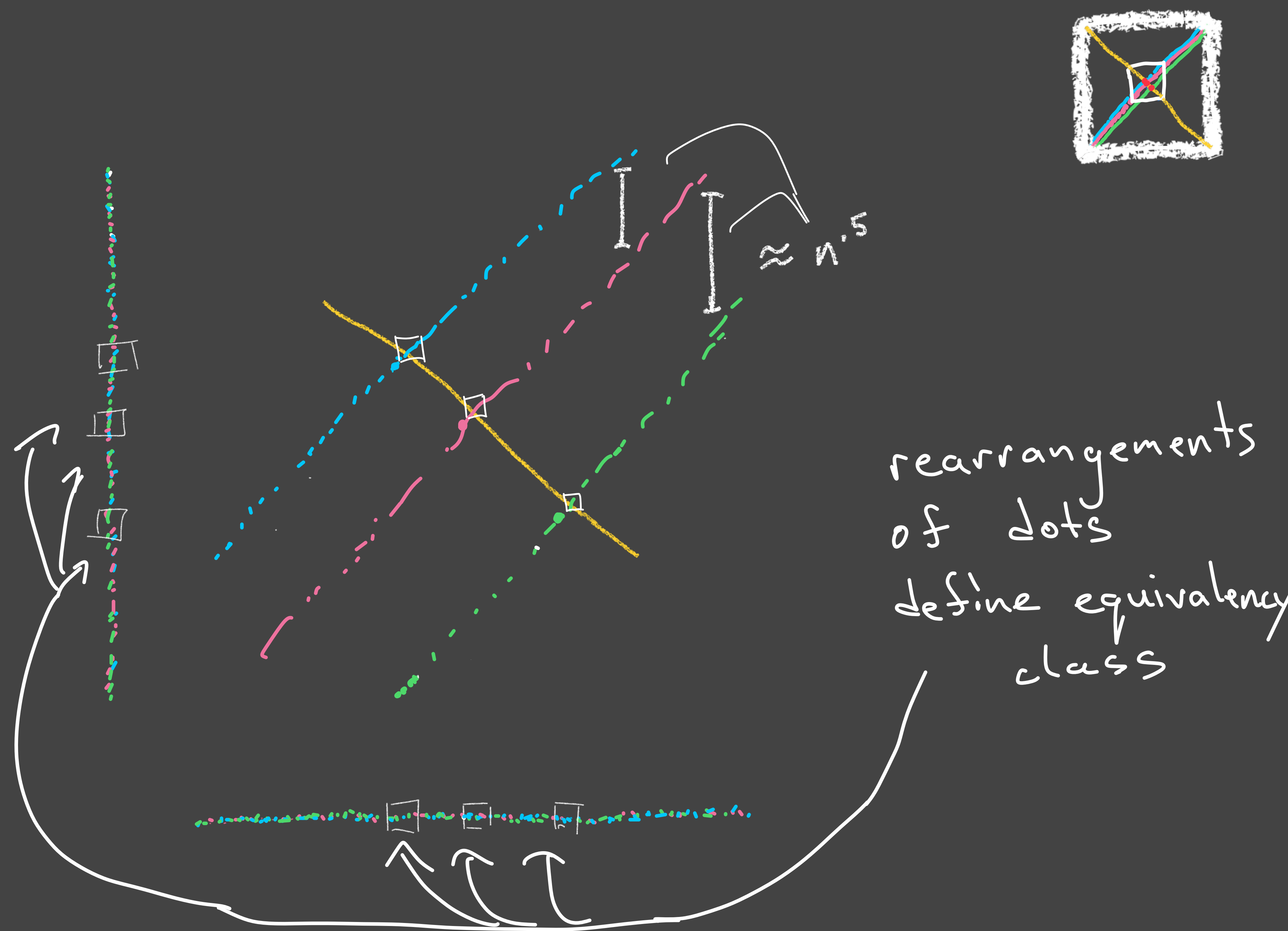


match up colors in increasing order to create permutation

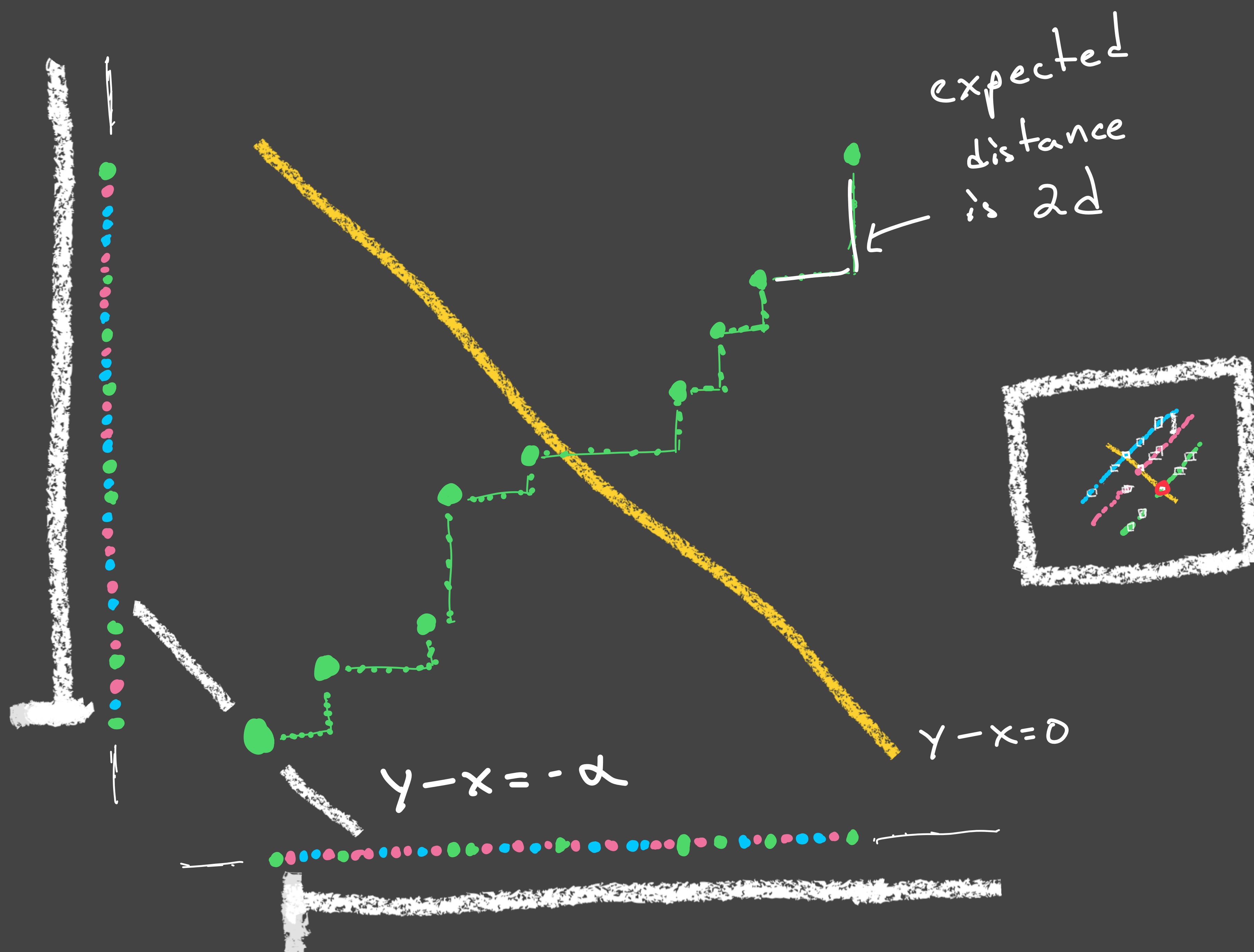
Anti-fixed pts of $Av_n((d+1)d\cdots 21)$ occur near center



# Near the center



# Near a potential crossing



# Near a potential crossing

$$P\left(\frac{\text{Hit}}{\xi}\right) = \frac{1}{2d}$$