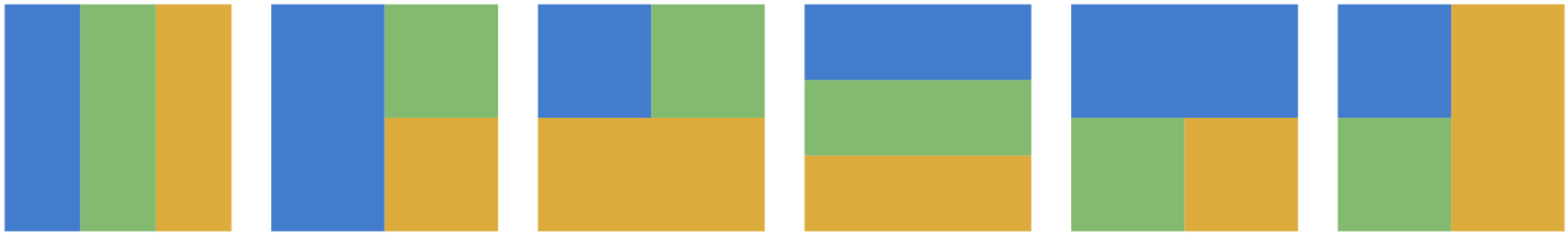
Permutation Patterns 2021

Pattern-avoiding rectangulations and permutations



Arturo Merino and Torsten Mütze

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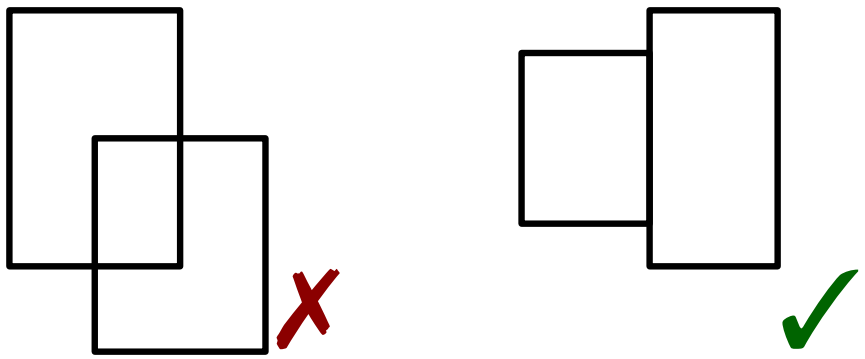

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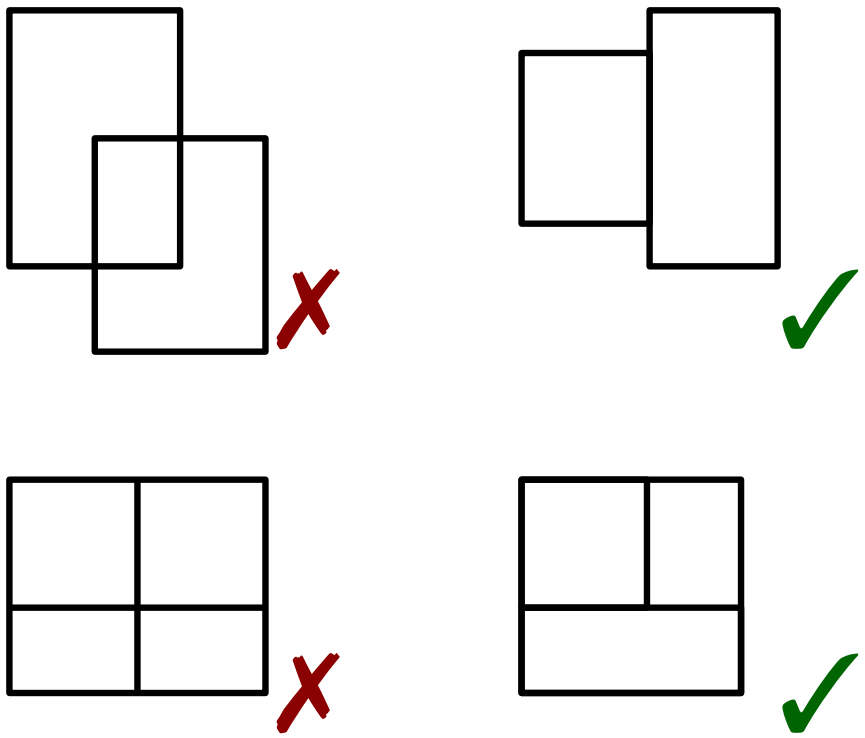

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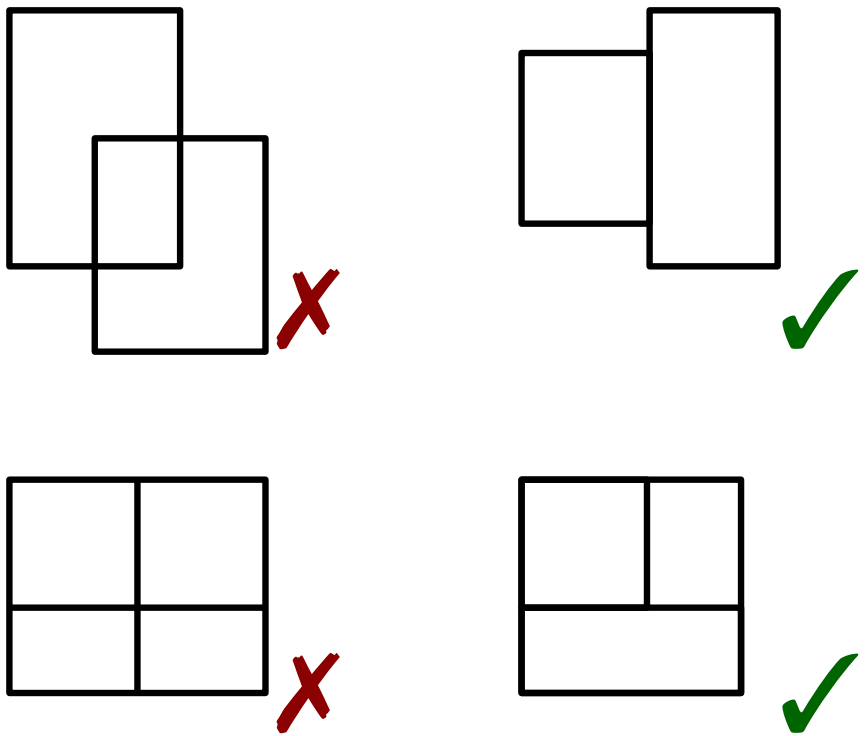

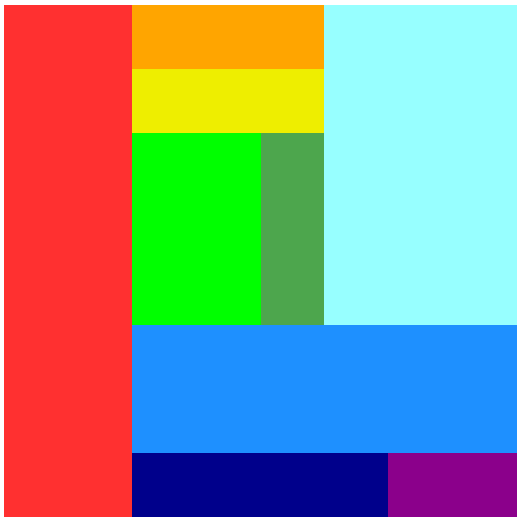

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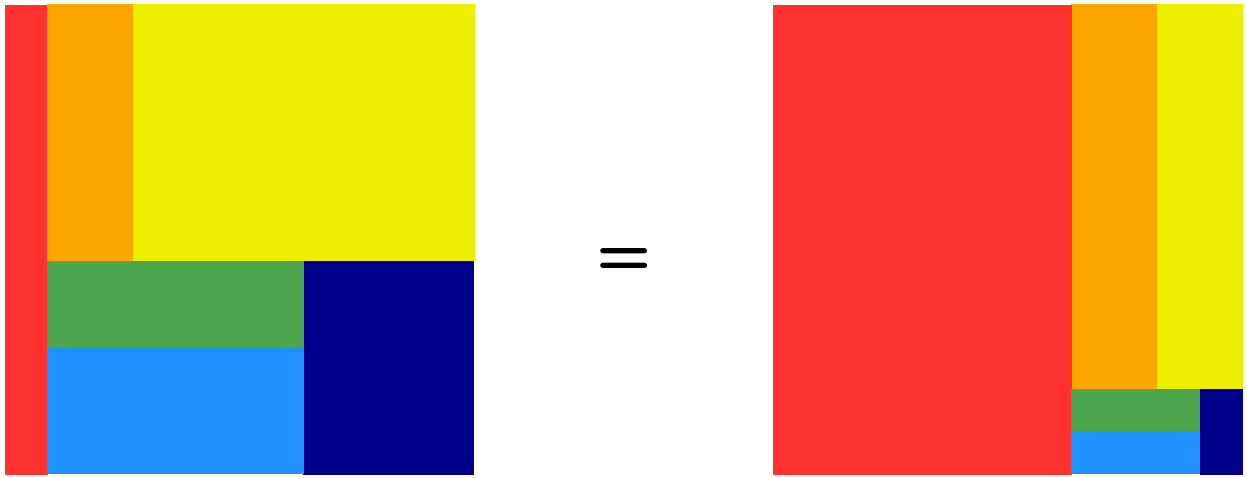

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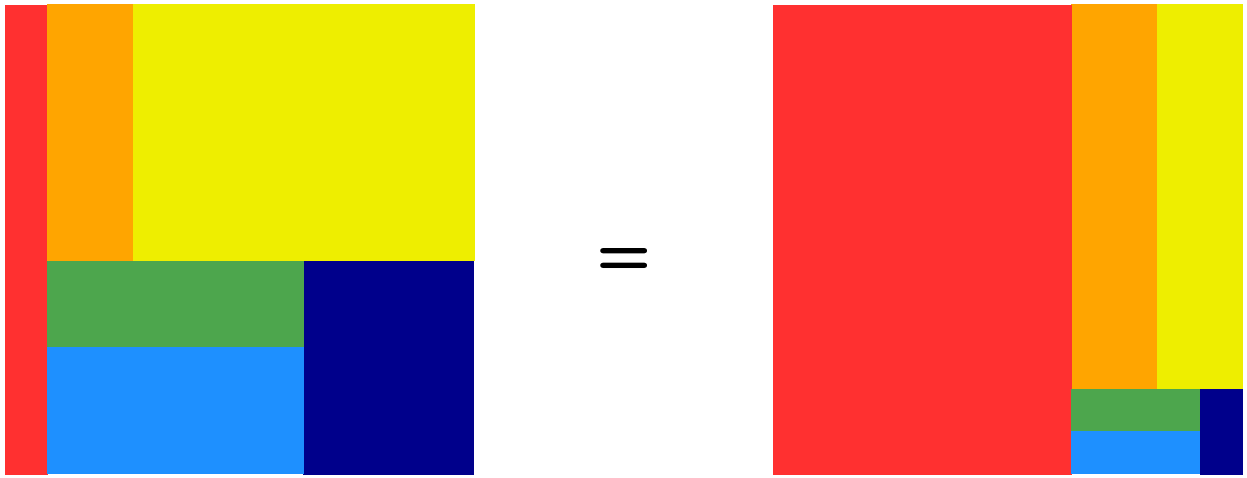

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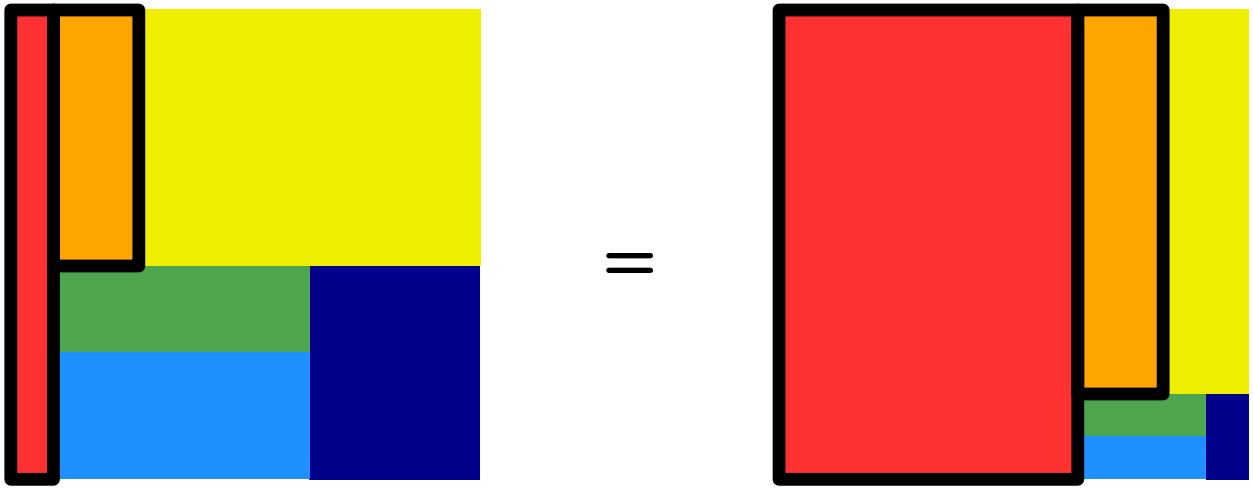

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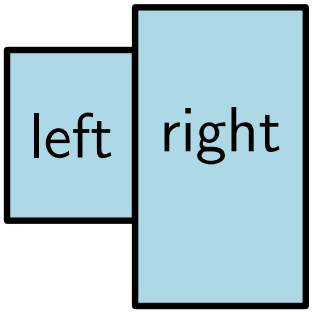

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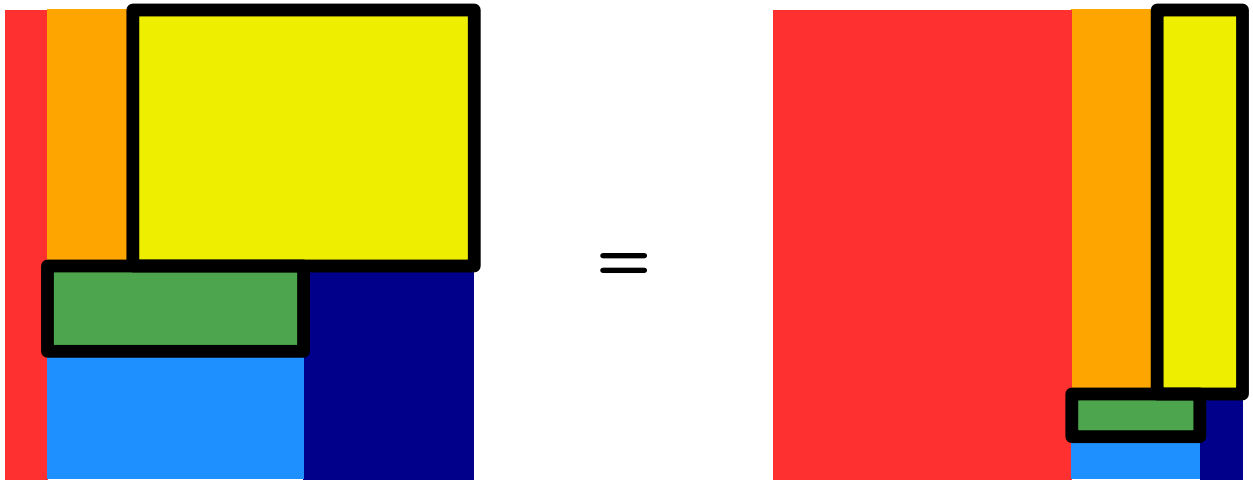

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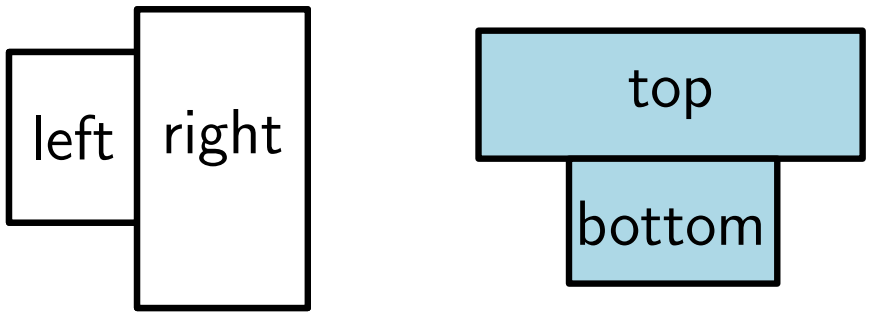

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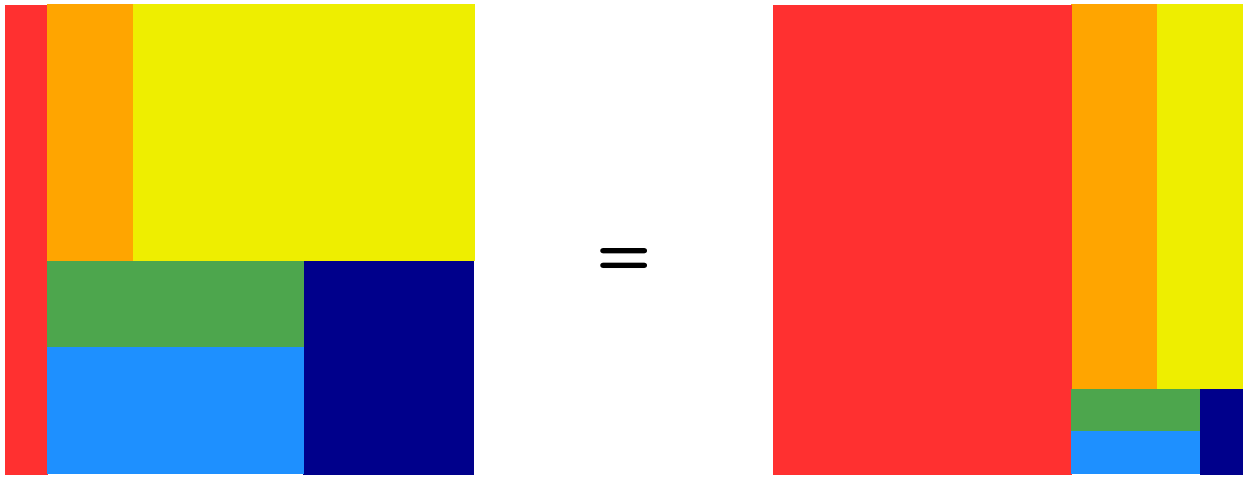

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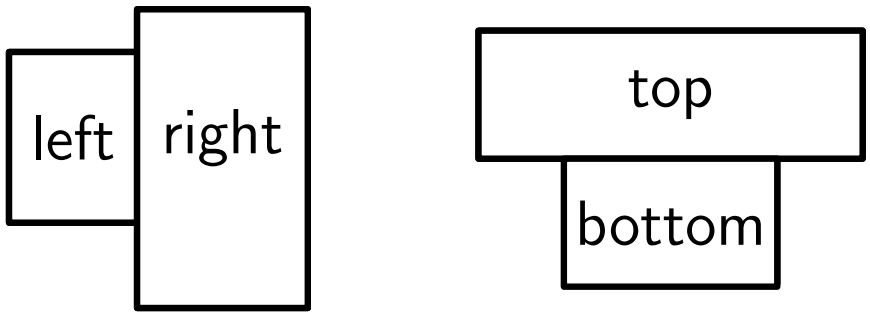

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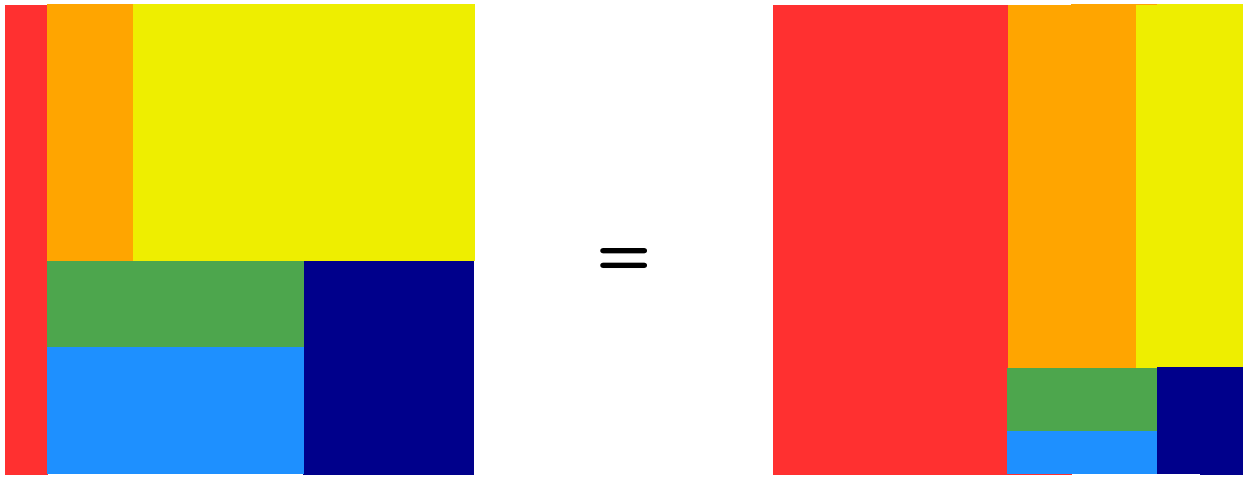

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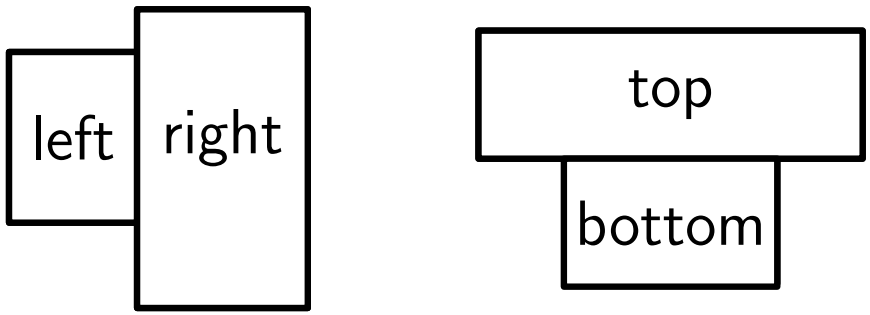

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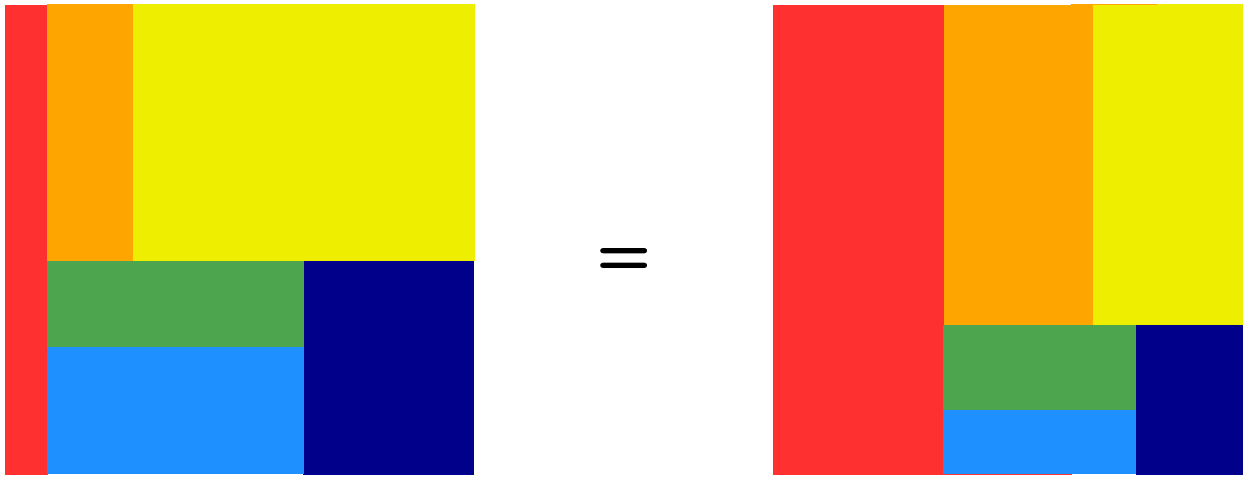

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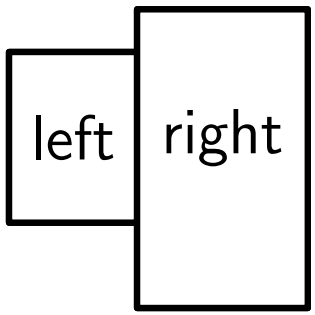

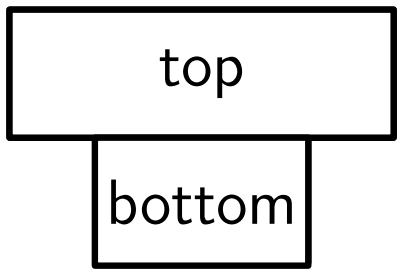

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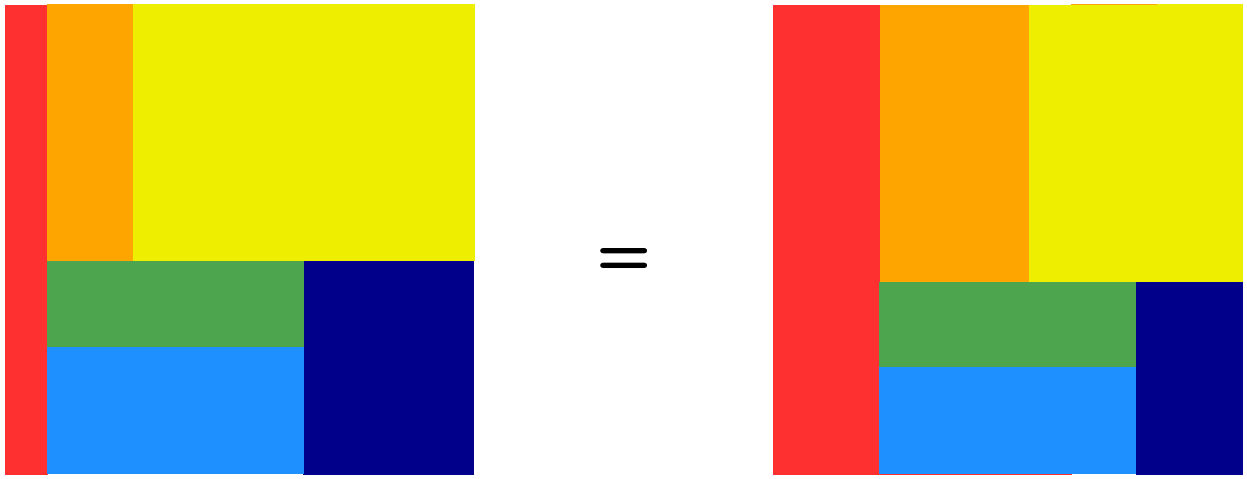

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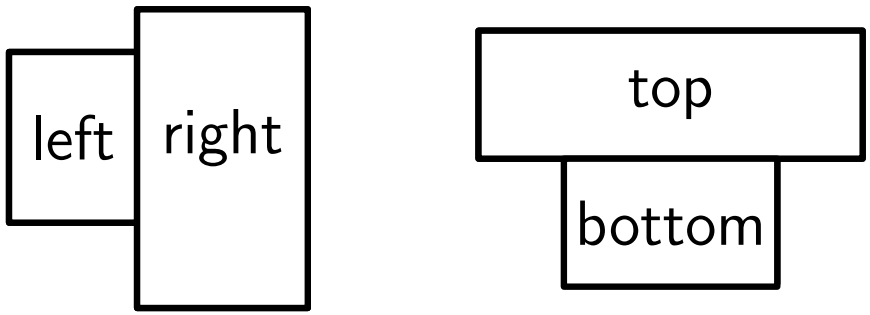

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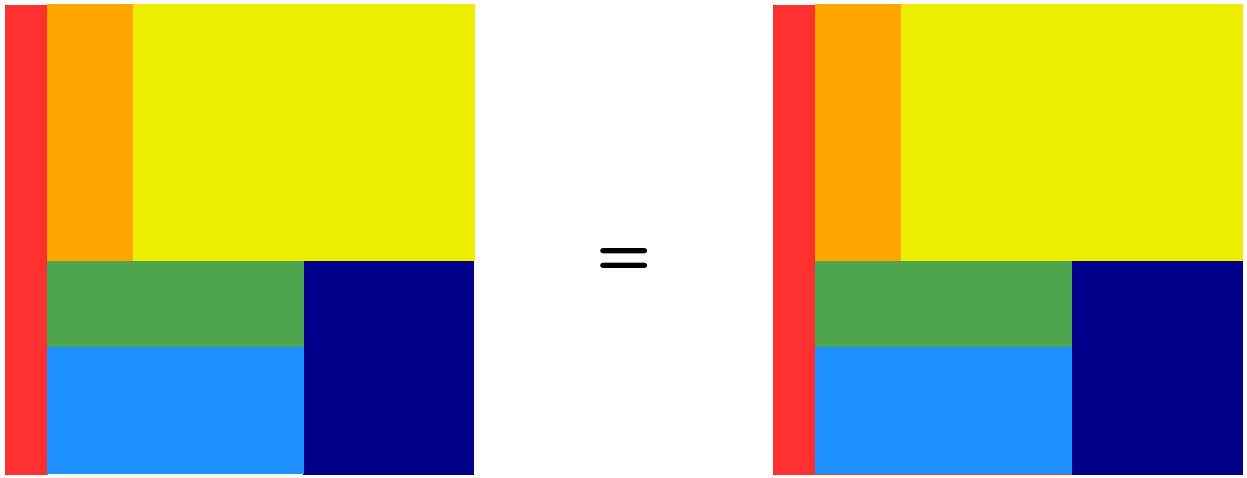

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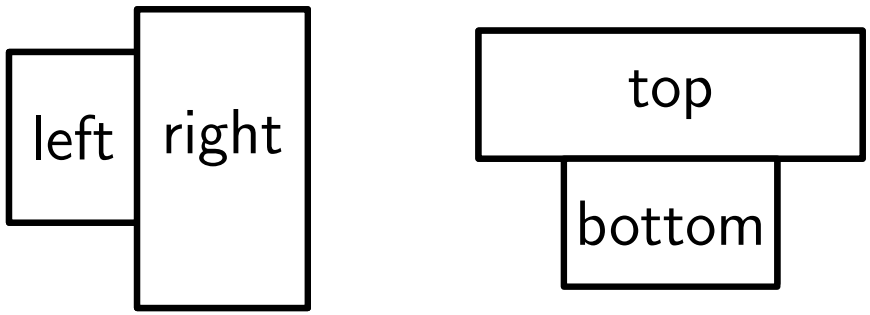

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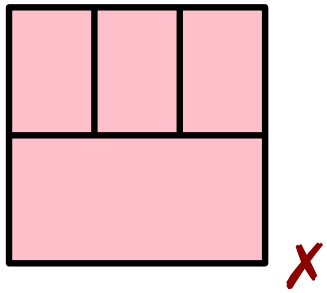

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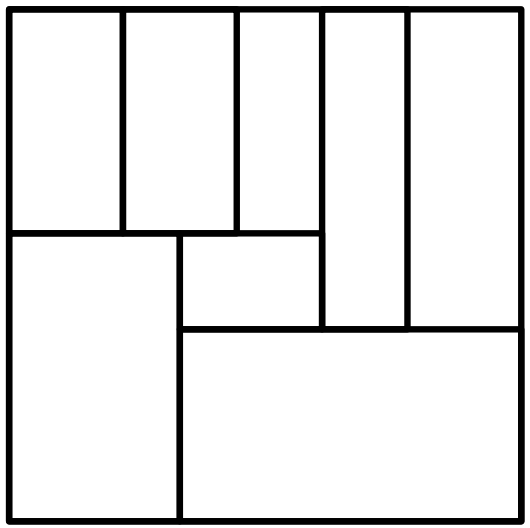

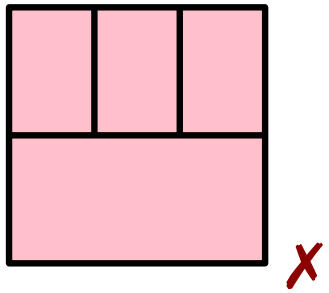

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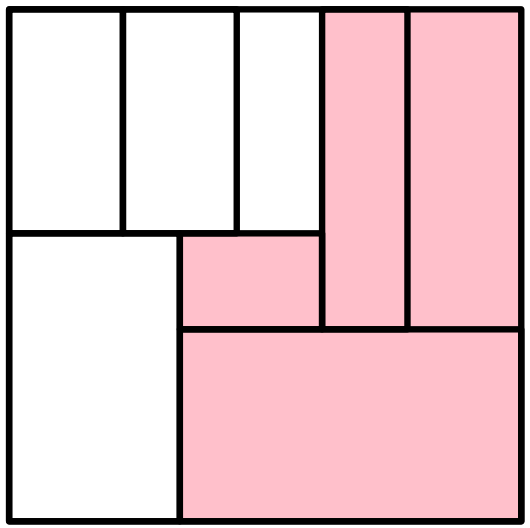

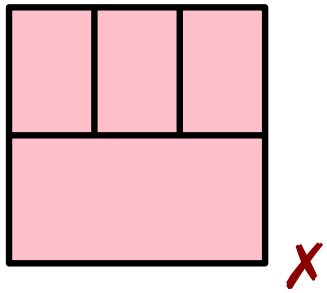

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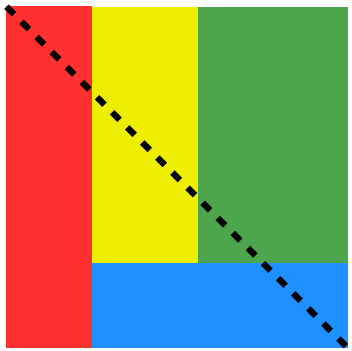

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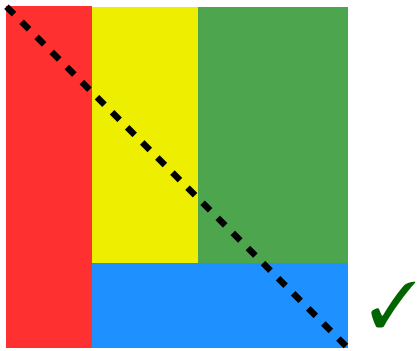

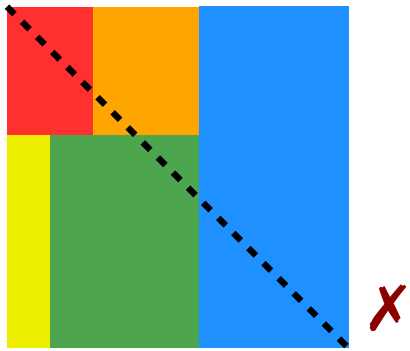

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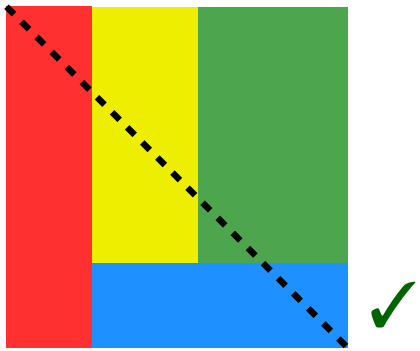
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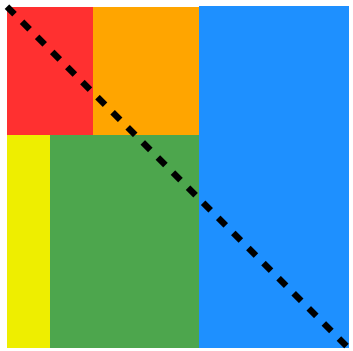
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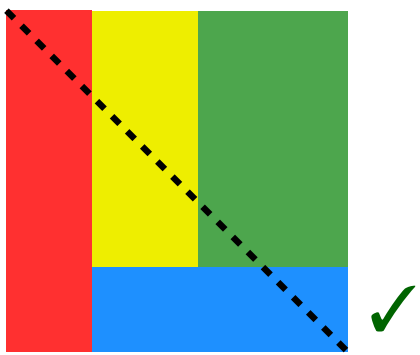
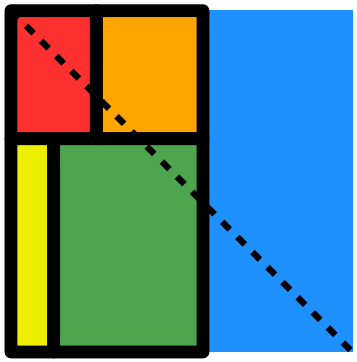
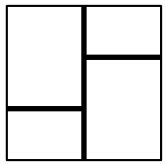
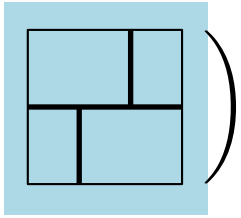



- $\mathrm{Diag}_n = \mathrm{Av}_n\Big($  $\wedge$  $\Big)$

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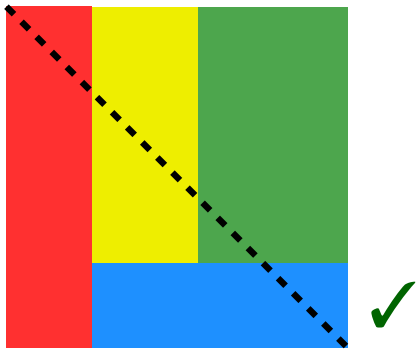
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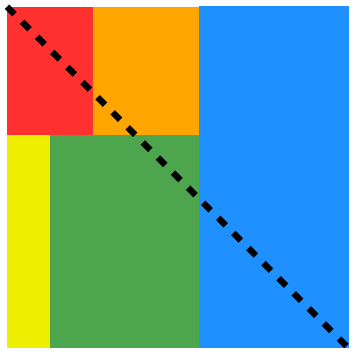
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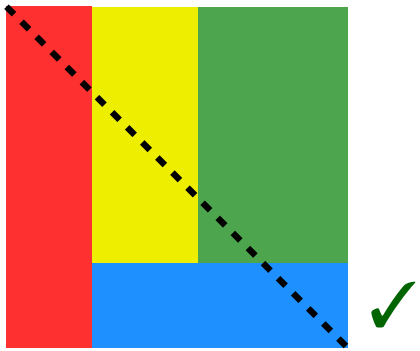
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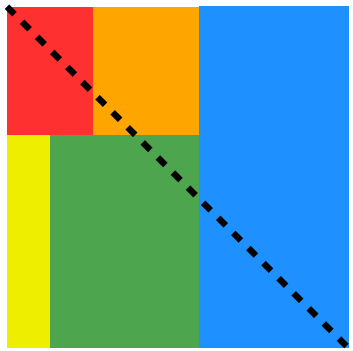
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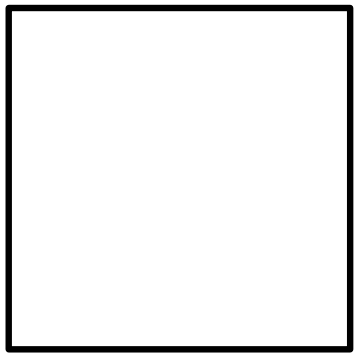

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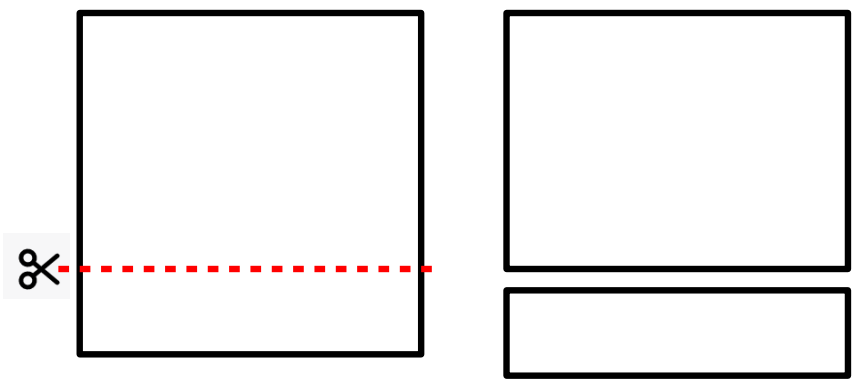

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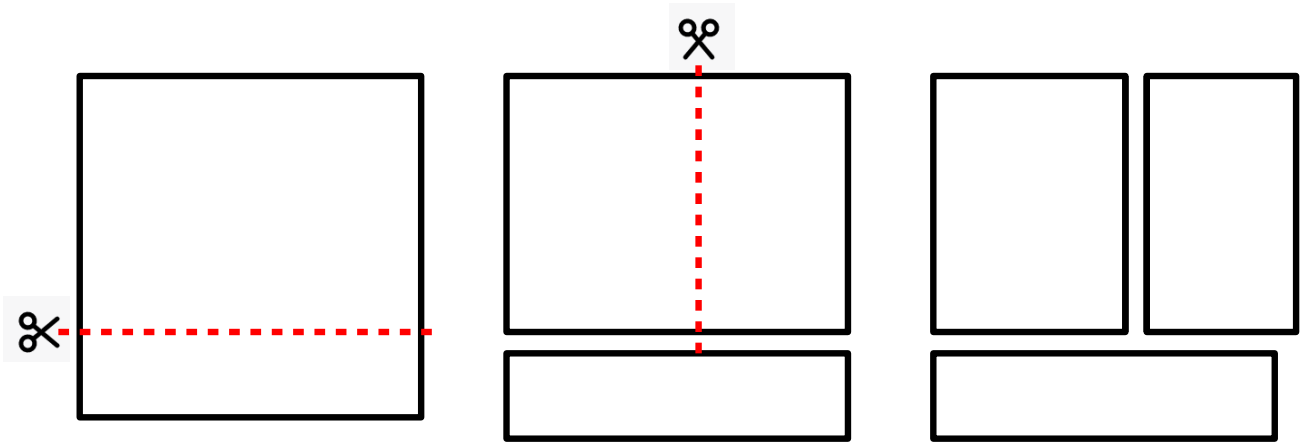

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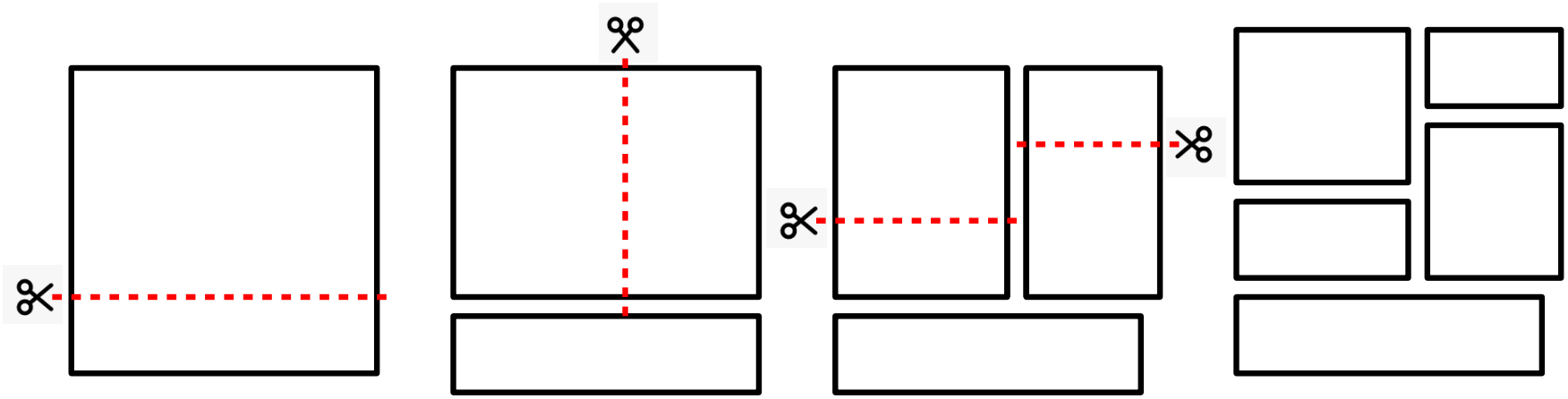

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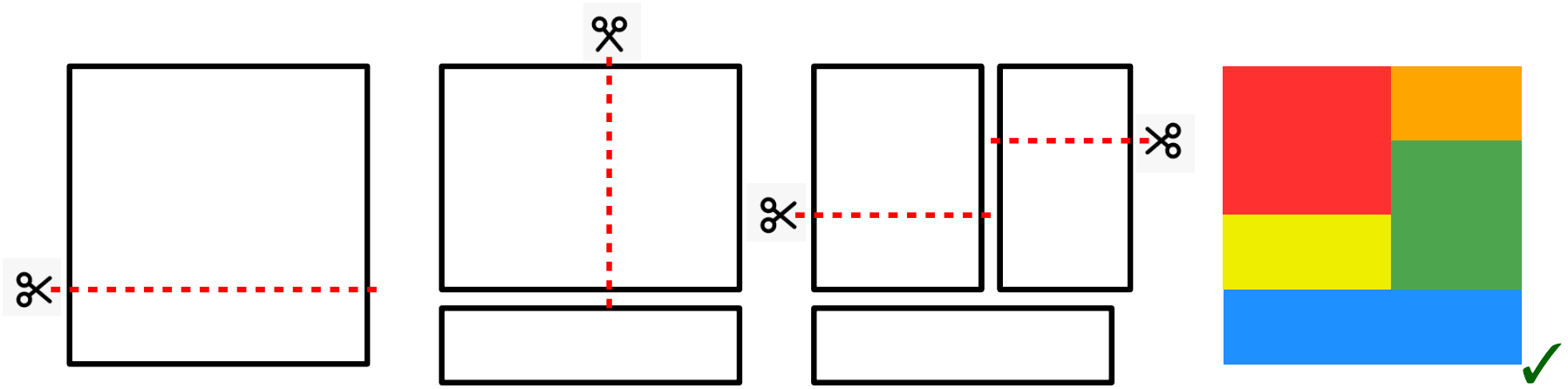

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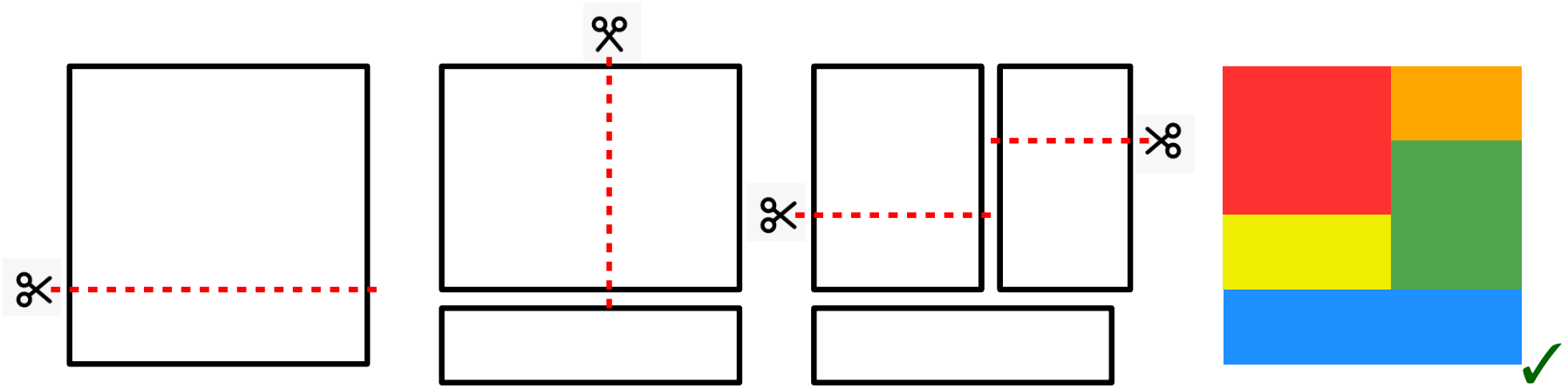

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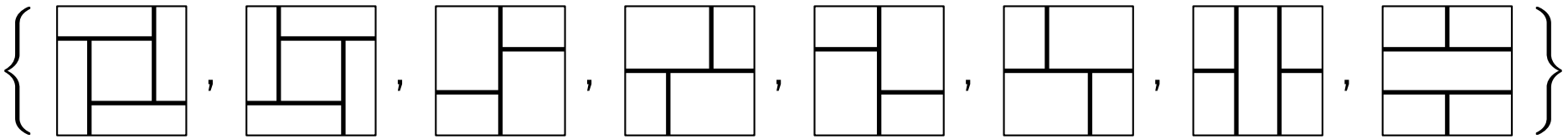
delay

Main results

Efficient generation for any combination of the following forbidden patterns



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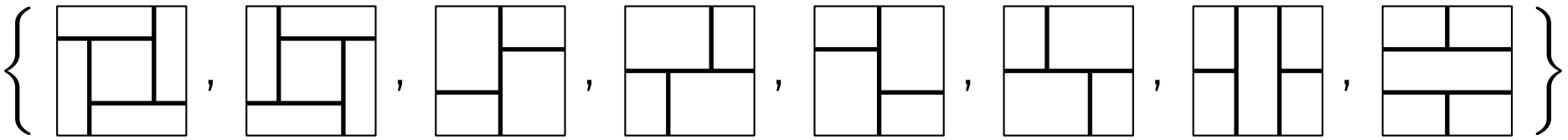

Family	Delay

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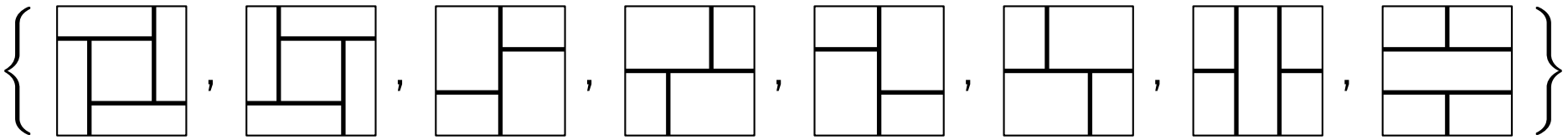

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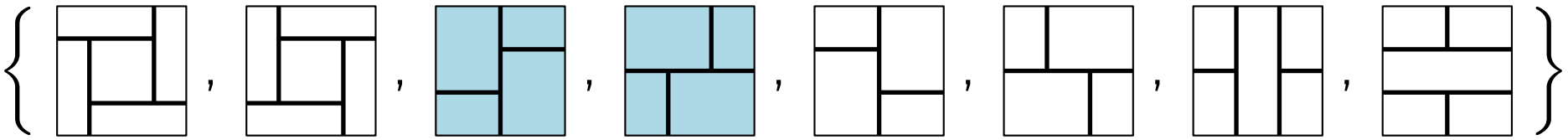

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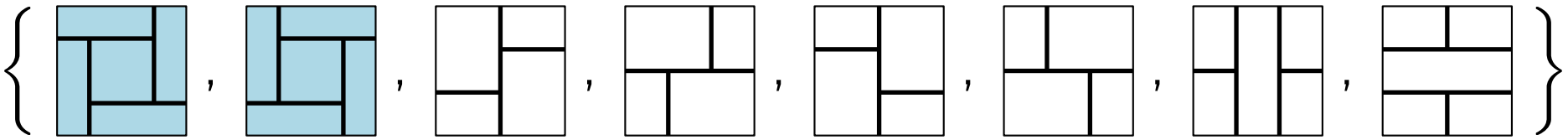

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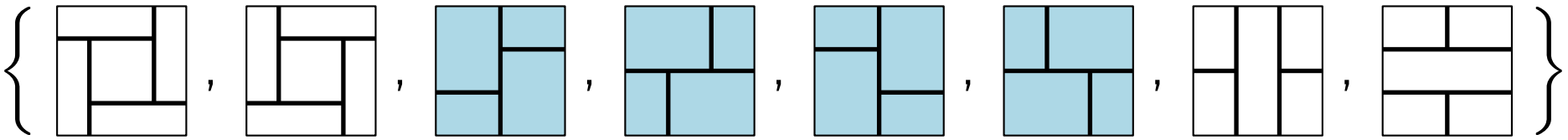

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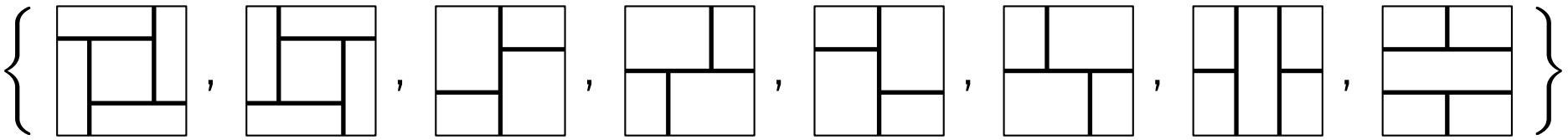

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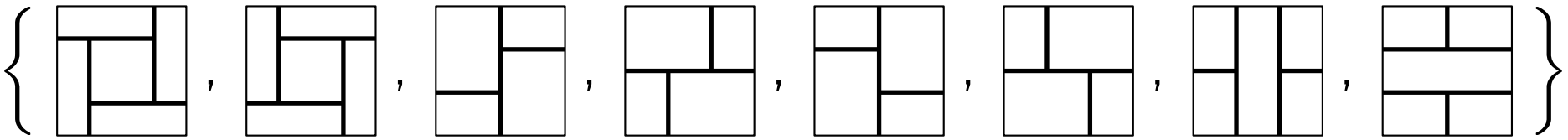

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C++ implementation on the Combinatorial Object Server:
`www.combos.org/rect`

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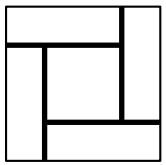
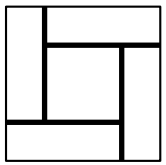
  - $\mathsf{av}_n\left(\right.$  $\wedge$  $\left. \right) = ?$

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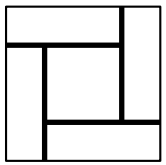
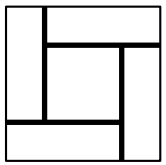
  - $\mathrm{av}_n\left(\right.$  $\wedge$  $\left.\right) = ?$

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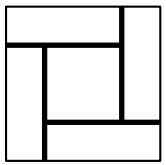
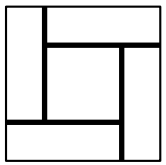
  - $\mathrm{av}_n\left(\right.$  $\wedge$  $\left.\right) = ?$

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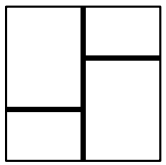
  - $\mathrm{av}_n\left(\right.$  $\left.\right) = \mathrm{av}_n(21\overline{3}54)?$

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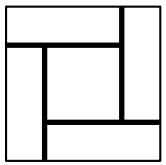
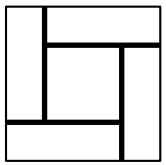
  - $\mathrm{av}_n\left(\right.$  $\wedge$  $\left.\right) = ?$

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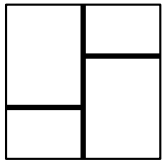
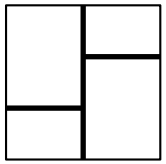
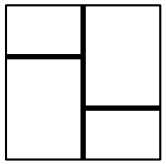
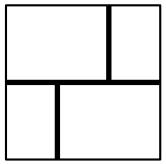
  - $\mathrm{av}_n\left(\right.$  $\left.\right) = \mathrm{av}_n(21\overline{3}54)?$
  - $\mathrm{av}_n\left(\right.$  $\wedge$  $\wedge$  $\left.\right) = \mathrm{av}_n(2\underline{41}3 \wedge 3\underline{14}2 \wedge 3\underline{41}2)?$

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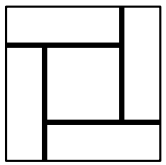
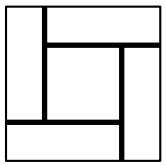
  - $\mathrm{av}_n\left(\right.$  $\wedge$  $\left.\right) = ?$

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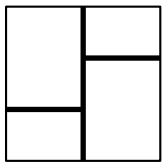
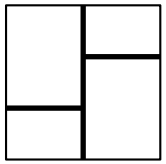
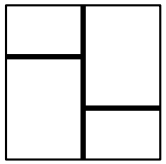
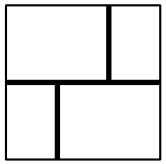
  - $\mathrm{av}_n\left(\right.$  $\left.\right) = \mathrm{av}_n(21\overline{3}54)?$
  - $\mathrm{av}_n\left(\right.$  $\wedge$  $\wedge$  $\left.\right) = \mathrm{av}_n(2\underline{41}3 \wedge 3\underline{14}2 \wedge 3\underline{41}2)?$

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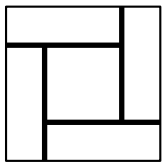
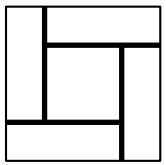
  - $\mathsf{av}_n\left(\right.$  $\wedge$  $\left.\right) = ?$

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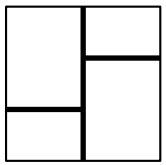
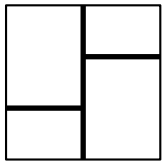
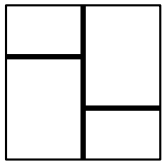
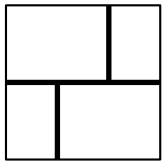
  - $\mathsf{av}_n\left(\right.$  $\left.\right) = \mathsf{av}_n(21\overline{3}54)$?
  - $\mathsf{av}_n\left(\right.$  $\wedge$  $\wedge$  $\left.\right) = \mathsf{av}_n(2\underline{41}3 \wedge 3\underline{14}2 \wedge 3\underline{41}2)$?

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