Spherical Schubert Varieties & Pattern Avoidance

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Def: The Flag variety Fl_n is the space of all complete flags $0 \subset V_1 \subset \cdots \subset V_n = \mathbb{C}^n$ with $\dim V_i = i$.

Def: For $w \in S_n$, the Schubert variety X_w is

$$X_w = \overline{BwB/B} \subseteq GL_n/B = Fl_n$$

where B is the group of upper triangular matrices.

Classical Results on Schubert Varieties & Pattern avoidance

Thm (Lakshmibai – Sandhya). X_w is smooth if and only if w avoids 4231 & 3412.

More results for:

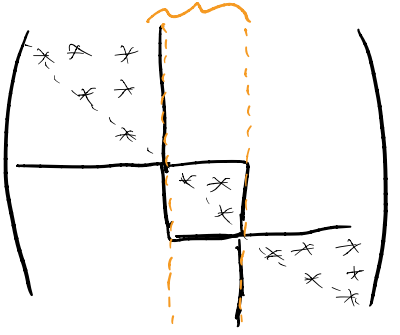
- when X_w is Gorenstein, toric, ...
- Properties of Schubert polynomials $\mathfrak{S}_w$
- Kazhdan – Lusztig polynomials
- Properties of intervals in Bruhat order & weak order

Spherical Schubert Varieties

Def (Hodges–Yong) X_w is (maximally) spherical if it contains a dense orbit of the group

non-descents of w^{-1}

$$B_w := \left\{ \left(\begin{array}{c} \\ \\ \\ \\ \end{array} \right) \in GL_n \right\}$$



Def (Hodges–Yong; Gao–Hodges–Yong)

$w \in S_n$ is spherical if $w_0(\mathrm{Des}(w^{-1}))\cdot w$ avoids 321 & 3412

Tenner: Boolean permutation

Main Result

Thm (Gao–Hodges–Yong) X_w is spherical if and only if w is spherical.

Conj (Hodges–Yong) w is spherical if and only if w avoids the following 21 patterns:

24531, 25314, 25341, 42531, 45231, 45312, 52314, 52341, 53124, 53142, 53412

34512, 34521, 35412, 35421, 45123, 45213, 54123, 54213, 54231

Thm The Conjecture holds.

Proof Comments

24531, 25314, 25341, 42531, 45231, 45312, 52314, 52341,
53124, 53142, 53412

$= \{ p = 5 \ldots 3 \ldots 1 \in S_5 \}$

no 2

no 4

34512, 34521, 35412, 35421, 45123, 45213, 54123, 54213, 54231,
45231, 53412

$= \{$ 4,5 appear before 1,2 & 3 is not between 2 & 4 $\}$

Thanks for Listening!