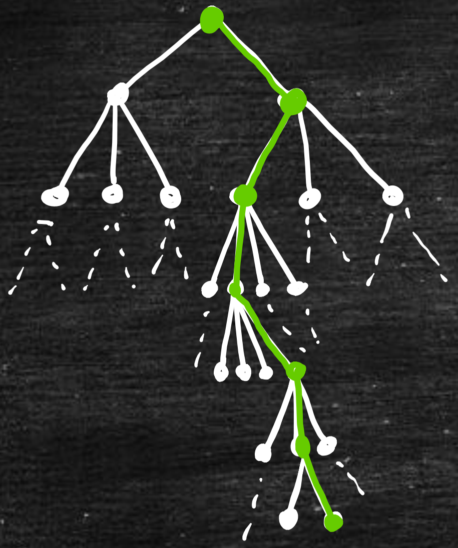

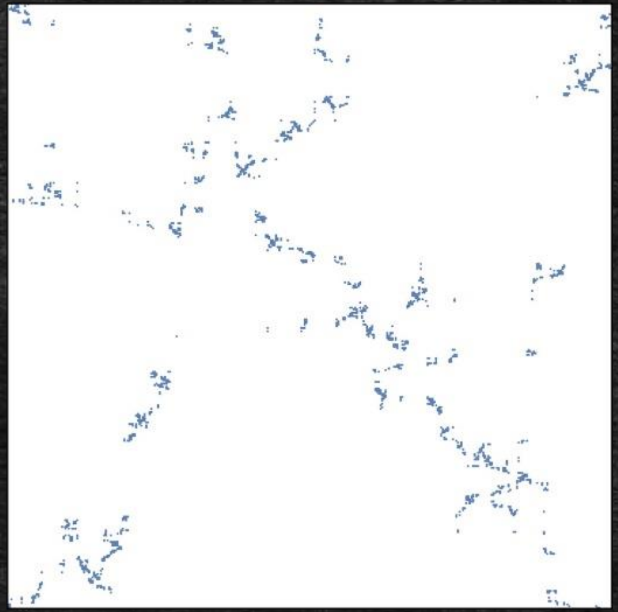

A probabilistic approach to generating trees

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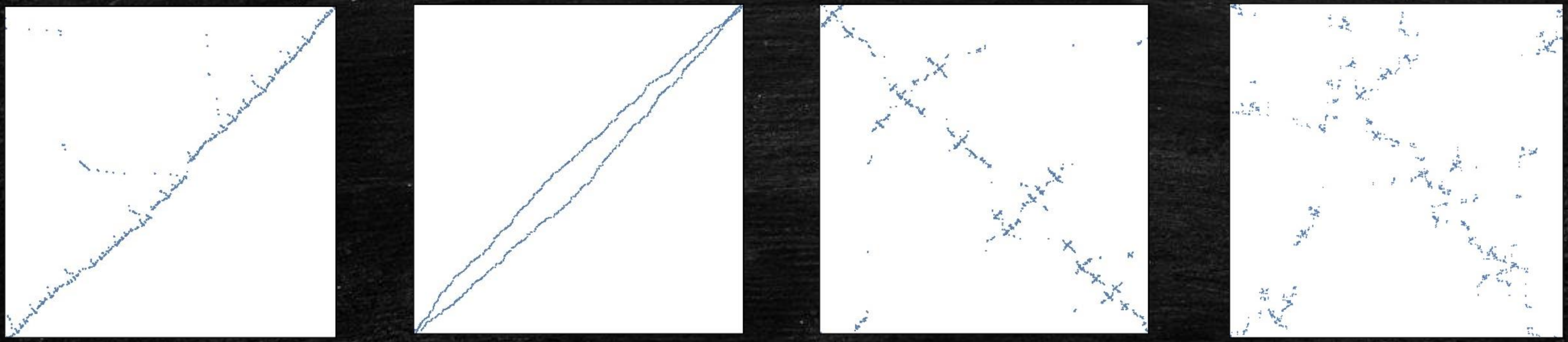
A REMARKABLY DIFFICULT QUESTION:

GIVEN A FAMILY OF PERMUTATIONS $\mathcal{C}$ SAMPLE A UNIFORMLY RANDOM PERMUTATION OF $\mathcal{C}$ OF LARGE SIZE.

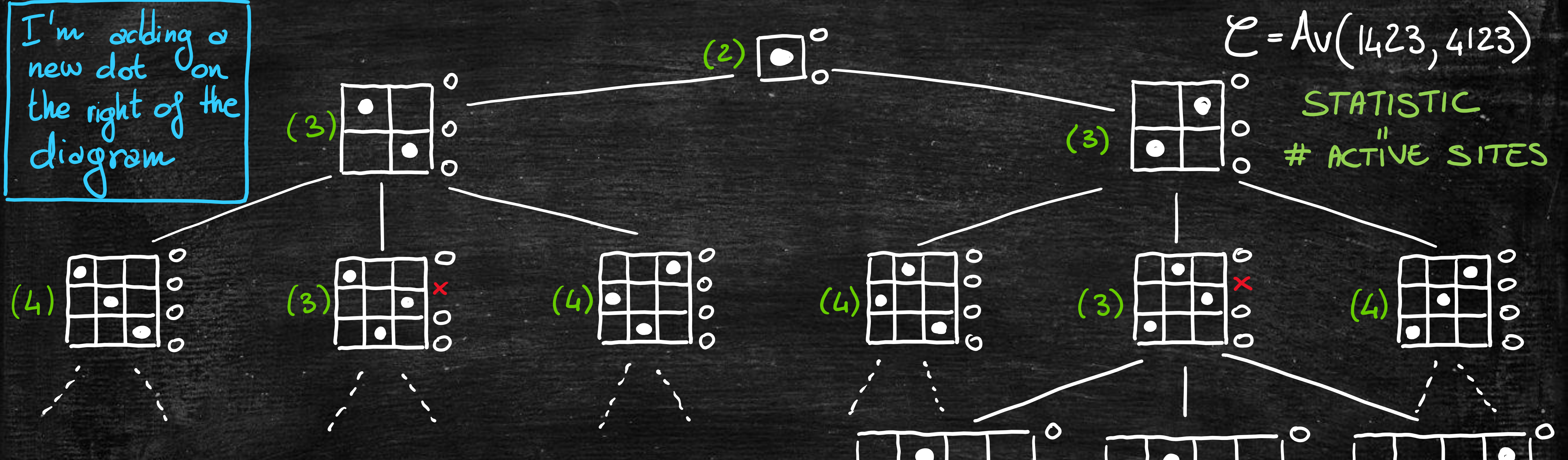


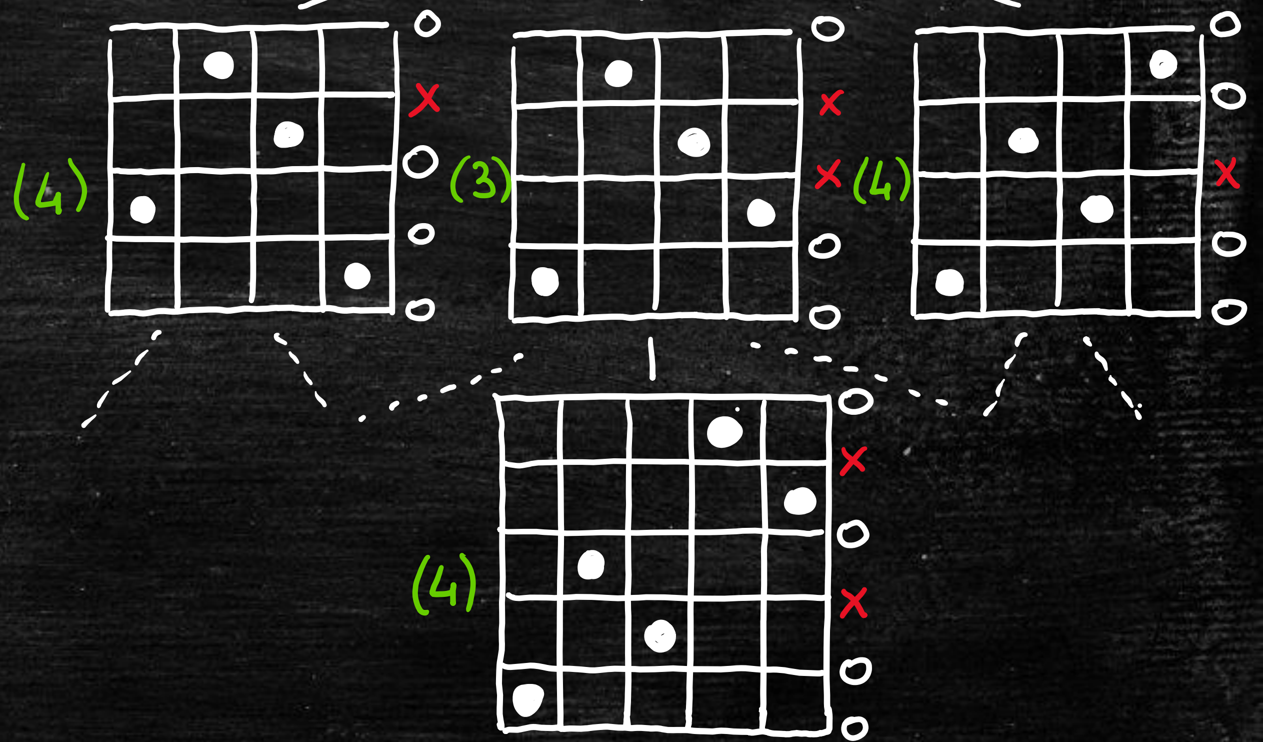
MESSAGE: Generating trees are <u>very</u> helpful!

What is a generating tree?

We give the def. of GENERATING TREE in the specific case of PERMUTATIONS but in general it can be defined for every combinatorial class $\mathcal{C}$.

Def: A GENERATING TREE for a class of permutations $\mathcal{C}$ is an infinite rooted tree whose vertices are the permutations of $\mathcal{C}$ (each appearing exactly once in the tree) and such that the objects of size n are at level n. The children of some permutation $\sigma \in \mathcal{C}$ correspond to the permutations obtained by adding a new dot to the diagram of σ.





KEY PROPERTY : <u>Succession Rule</u>

- A S-valued statistic on $\mathcal{C}$ whose value determines the number of children in the generating tree and the values of the statistic for the children.

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- The corresponding SUCCESSION RULE is then given by the label λ of the root and, for any label k, the labels

$$e_1(k), \dots, e_h(k)$$

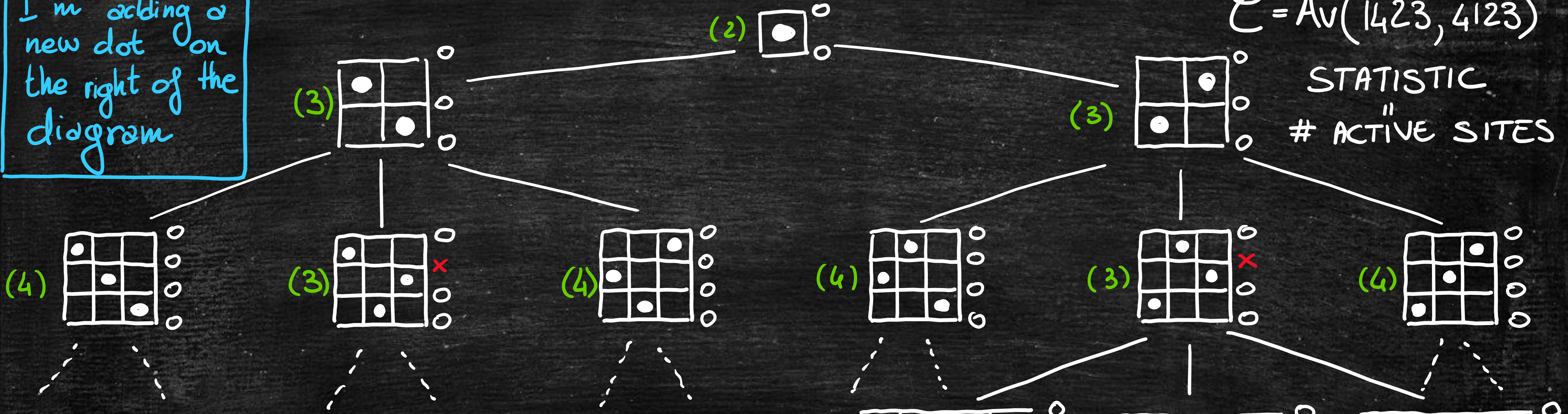
of the h children of an object labeled by k.

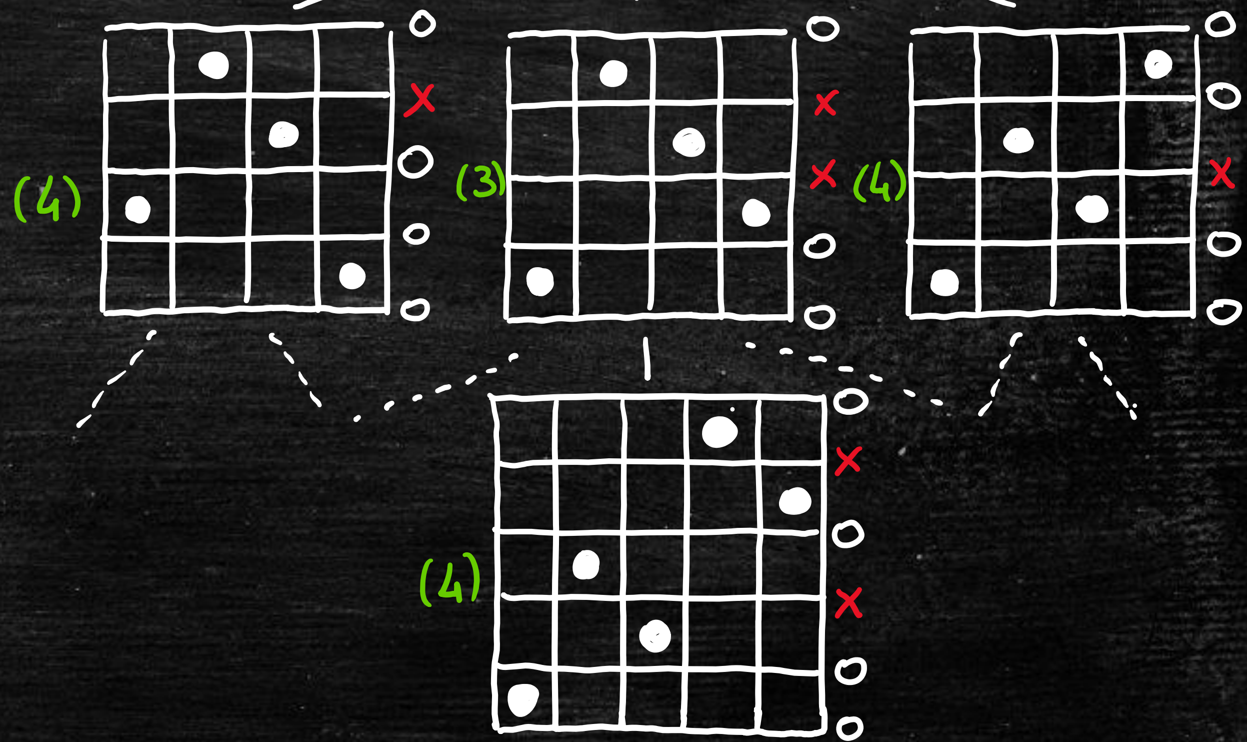
$$\begin{cases} \text{Root label: } \lambda \\ (k) \longmapsto (e_1(k)), \dots, (e_h(k)) \end{cases}$$

I'm adding a new dot on the right of the diagram

$\mathcal{C} = Av(1423, 4123)$

STATISTIC = # ACTIVE SITES





KEY PROPERTY : Succession Rule

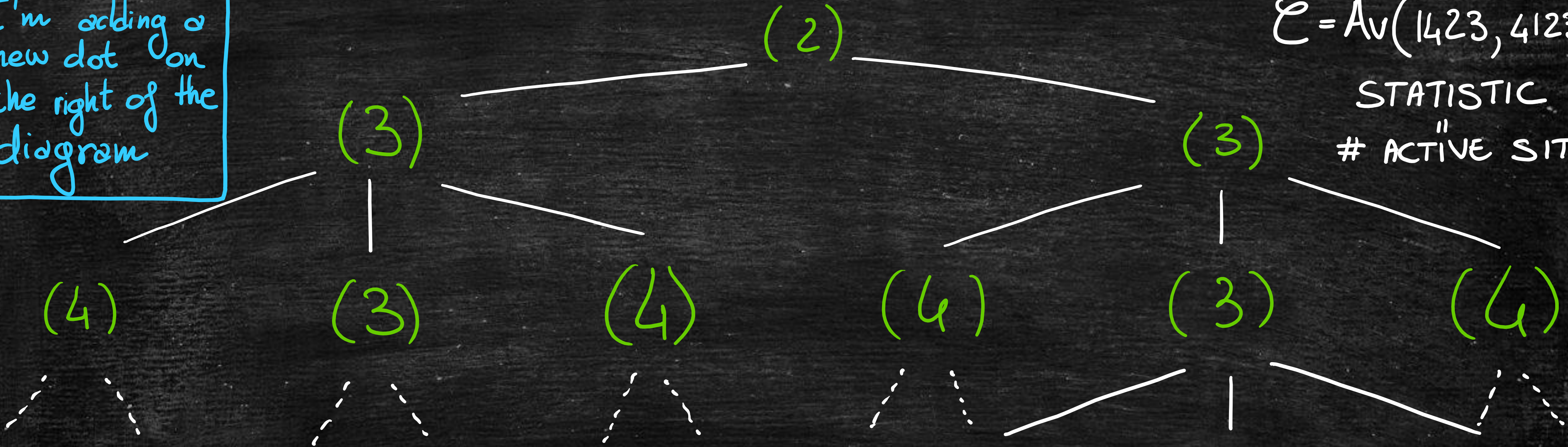
$$\begin{cases} \text{Root label: } (2) \\ (k) \longmapsto (k+1), (3), (4), \ldots, (k+1) \end{cases}$$

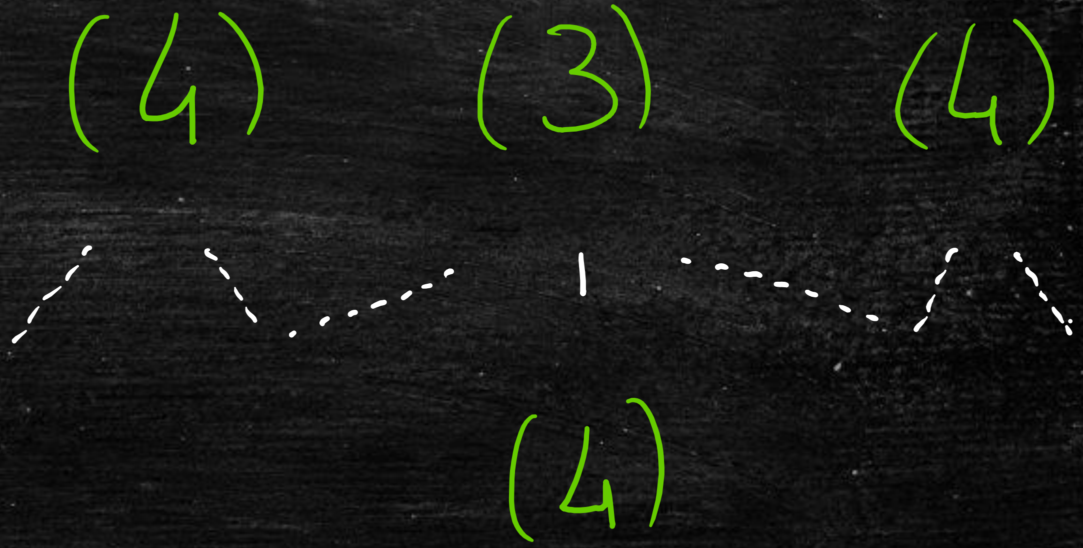
(Proved by Kremer '00)

I'm adding a new dot on the right of the diagram

$\mathcal{C} = Av(1423, 4123)$

STATISTIC $=$ # ACTIVE SITES





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A bijection between walks and permutations

$\{$Permutations of size n in $\mathcal{C}\}$ $\xleftrightarrow{\text{bij}}$ $\{$Paths in the generating tree ending at level $n\}$

$\updownarrow$ bij

$\{$Sequences of n labels starting at 1 and consistent with the succession rule$\}$

IMPORTANT MESSAGE:

It is very SIMPLE to sample uniform sequences of n labels starting at 1 and consistent with the succession rule

$\Rightarrow$ SIMPLE to sample uniform permutations of size n in $\mathcal{C}$.

Type of results that one can then prove:

Theorem (Borga 2020)

Let $\mathcal{C}$ be one of the following families of permutations:

$$\mathrm{Av}(123),\ \mathrm{Av}(132),\ \mathrm{Av}(1423,4123),\ \mathrm{Av}(1234,2134),\ \mathrm{Av}(1324,3124)$$

$$\mathrm{Av}(2314,3214),\ \mathrm{Av}(2413,4213),\ \mathrm{Av}(3412,4312),\ \mathrm{Av}(213,\bar{2}\,\underline{31}),\ \mathrm{Av}(213,\bar{2}^{\circ}\underline{31})$$

For all $n\in\mathbb{N}$, let σ_n be a uniform random permutation in $\mathcal{C}$ of size n.

The for all $\pi\in\mathcal{S}$, $\exists\ \mu_\pi,\gamma_\pi^2\in\mathbb{R}$, such that

$$\frac{\text{c-occ}(\pi,\sigma_n)-n\cdot\mu_\pi}{\sqrt{n}}\xrightarrow{d}\mathcal{N}(0,\gamma_\pi^2)$$

PLEASE, SEND ME AN EMAIL, IF YOU KNOW A FAMILY OF (CLASSICAL) PATTERN-AVOIDING PERMUTATIONS ENCODED by A GENERATING TREE WITH LABELS IN $\mathbb{Z}^2_{>0}$

THANK YOU!