Pattern Avoidance and Fiber Bundle Structures on Schubert Varieties

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(This talk is based on joint work with Edward Richmond.)

MSC2000: 05E15, 14M15

Permutations are known to index type A Schubert varieties. A question one can ask regarding a specific Schubert variety is whether it is an iterated fiber bundle of Grassmannian Schubert varieties, that is, if it has a complete parabolic bundle structure. We have found that a Schubert variety has a complete parabolic bundle structure if and only if its associated permutation avoids the patterns 3412, 52341, and 635241. We find this by first identifying when the standard projection from the Schubert variety in the complete flag variety to the Schubert variety in the Grassmannian is a fiber bundle using what we have called "split pattern avoidance". In this talk, I will demonstrate how we were able to move from a characterization of this projection in terms of the support and left descents of the permutation's parabolic decomposition to one that applies split pattern avoidance. I will also give a flavor of the proof of how the three patterns mentioned above determine whether a Schubert variety has a complete parabolic bundle structure.

A simple proof of Shamir's conjecture

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(This talk is based on joint work with Julia Böttcher, Ewan Davies, Matthew Jenssen, Yoshiharu Kohayakawa, Barnaby Roberts.)

MSC2000: 05C80

It is well known (and easy to show) that the threshold for a perfect matching in the binomial random graph $G(n, p)$ is $p = \Theta\left(\frac{\log n}{n}\right)$, coinciding with the threshold for every vertex to be in an edge (and much more is known). However when one moves to hypergraphs, life becomes difficult. In 1983, Schmidt and Shamir gave the first non-trivial values of p such that the binomial random 3-uniform hypergraph $G^{(3)}(n, p)$ contains a perfect matching with high probability, and they observed that the threshold is $\omega\left(\frac{\log n}{n^2}\right)$, since below this point some vertices of $G^{(3)}(n, p)$ are not in any edges. They conjectured that in fact the threshold is $\Theta\left(\frac{\log n}{n^2}\right)$, which was finally resolved in a notoriously difficult 2008 paper of Johansson, Kahn and Vu. I will give a simpler proof.

A new forbidden subgraph for 5-contractible edges

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MSC2000: 05C40

We deal with finite undirected graphs with neither self-loops nor multiple edges. For a given graph H, a graph G is said to be H*-off* if G has no subgraph isomorphic to H. Note that H is not necessary induced. An edge e of a k-connected graph is said to be k*-contractible* if the contraction of the edge results in a k-connected graph. A fixed graph H is said to be a *forbidden subgraph for* k*-contractible edges* if every H-off k-connected graph with sufficiently large order has a k-contractible edge.

Thomassen [3] pointed out that K_3 is a forbidden subgraph for k-contractible edges. The following result due to Ando et al. [1] is an extension of the above Thomassen's result.

Theorem 1. *Every* $(K_1 + 2K_2)$*-off* k*-connected graph has a* k*-contractible edge.*

Hereafter we consider 5-connected graphs. We call the left graph $K_1 + (P_3 \cup K_2)$ and the right graph in the following Fig. 1, F(*fish*) and F_f(*fish with fin*), respectively.

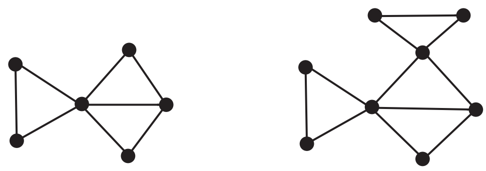

Fig. 1. F and F_f

Kawarabayashi [2] proved F is a forbidden subgraph for 5-contractible edges. Since F contains a $K_1 + 2K_2$, this is an extension of Theorem 1 for 5-connected case.

Theorem 2. *Every* F*-off* 5*-connected graph has a* 5*-contractible edge.*

Our main result is that following which is an extension of Theorem 2.

Theorem 3. *Every* F_f*-off* 5*-connected graph has a* 5*-contractible edge.*

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[2] K. Kawarabayashi, Contractible edges and triangles in k-connected graphs, *J. Combin. Theory* (B) **85** (2002), 207–221.

[3] C. Thomassen, Non-separating cycles in k-connected graphs, *J. Graph Theory* **5** (1981), 351–354.

Planar graphs - improper colorings and coverings with induced forests

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(This talk is based on joint work with D. Goncalves, J. Rollin, T. Ueckerdt, and P. Weiner.)

MSC2000: 05C15, 05C10, 05C70, 05C38

What happens when the vertices of a planar graph are colored with less than four colors? We know that the coloring might be improper, i.e., contain adjacent vertices of the same color. Can one nevertheless make sure that the color classes induce simple enough graphs, such as vertex-disjoint unions of short paths? We shall present the history of the problem and some recent progress towards its solution.

It is well known that the edge set of any planar graph could be covered by three forests. What if the covering forests are required to be induced? What if these forests are required to have large components? We show that the number of such covering forests is bounded by a constant for planar graphs and for graphs from other large families.

Compatible Cycle Decomposition of bad K_5-minor-free eulerian graphs

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(This talk is based on joint work with Herbert Fleischner, Cun-Quan Zhang, and Zhang Zhang.)

MSC2000: 05C38, 05C45, 05C70, 05C83

Let G be an eulerian graph. For each vertex $v \in V(G)$, let $\mathcal{T}(v)$ be a partition of the edges incident with v into $2-$subsets and set $\mathcal{T} = \cup_{v \in V(G)} \mathcal{T}(v)$, called a transition system of G. A transition system $\mathcal{T}$ of G is admissible if $|T \cap F| \leq \frac{1}{2}|F|$ for every $T \in \mathcal{T}$ and every edge cut F of G. A cycle decomposition $\mathcal{C}$ of G is called a compatible cycle decomposition (CCD for short) of $(G, \mathcal{T})$ if $|E(C) \cap T| \leq 1$ for every cycle $C \in \mathcal{C}$ and every $T \in \mathcal{T}$. H. Fleischner proved that if G is planar, then for every admissible transition system $\mathcal{T}$ of G, $(G, \mathcal{T})$ has a CCD. G. Fan and C.-Q. Zhang (2000, J. Combin. Theory Ser. **B 78**, 1-23) showed that this result is also true for K_5-minor-free graphs. We generalize this result to all eulerian graphs that do not contain a special type of K_5-minor which is called a bad K_5-minor. To this purpose, we characterise the $4-$regular transitioned graphs $(G, \mathcal{T})$ without bad K_5-minor in which every CCD of $(G, \mathcal{T})$ is a pair of hamiltonian cycles.

A database of distance-regular graphs

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(This talk is based on joint work with A. D. M. Jackson, C. H. Weir.)

MSC2000: 05E30

A graph with diameter d is distance-regular, if for any pair of vertices u, v at distance i (with $1 \leq i \leq d$), the number of neighbours of v at distances $i-1$, i and $i+1$ from u depends only on i, and not on the choice of u and v. Over the past few years, I have been gathering a collection of distance-regular graphs, which I felt may be useful to others. So, with the help of some grant funding, I acquired the domain name `www.distanceregular.org`, and employed some students to assist with putting a catalogue online. In this talk, I will give some background material, a brief demonstration of the site itself, and what may be available in the future.

Plenary lecture

Monday 11.15 Main Auditorium

Relations among partitions

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MSC2000: 05B30

Combinatorialists often consider a balanced incomplete-block design to consist of a set of points, a set of blocks, and an incidence relation between them which satsifies certain conditions. To a statistician, such a design is a set of experimental units with two partitions, one into blocks and the other into treatments; it is the relation between these two partitions which gives the design its properties. The most common binary relations between partitions that occur in statistics are refinement, orthgonality and balance. When there are more than two partitions, the binary relations may not suffice to give all the properties of the system. I shall survey work in this area, including designs such as double Youden rectangles and triple arrays.

The emergence of the square of a Hamilton cycle in random geometric graphs

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(This talk is based on joint work with Mark Walters.)

MSC2000: 05C80

The Gilbert model is a random geometric graph defined by placing n points at random in a square and then joining pairs of points if they are within some distance r of each other. It is natural to ask how large r must be before familiar graph properties (such as connectedness, Hamiltonicity, et cetera) typically occur in this model. Answering a question of Penrose, it was shown by Balogh, Bollobás, Krivelevich, Müller and Walters [1] that the 'obstruction' to Hamiltonicity is the presence of a vertex of degree at most one. More precisely, they showed that for a typical point set, as we increase r, the graph becomes Hamiltonian exactly once the graph has minimum degree at least two.

We consider the property of containing the square of a Hamilton cycle, showing that the obstruction is some vertex not occurring as the root of a particular rooted graph on five vertices – not a minimum degree condition. Perhaps surprisingly, unlike the binomial random graph, which is not typically square Hamiltonian until it has minimum degree $\Omega(\sqrt{n})$, a random geometric graph is typically square Hamiltonian whilst it still has bounded minimum degree.

[1] J. Balogh, B. Bollobás, M. Krivelevich, T. Müller, and M. Walters. Hamilton Cycles in Random Geometric Graphs. *The Annals of Applied Probability*, 1053–1072, 2011.

Creating a Virtual Combinatorist

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(This talk is based on joint work with Michael Albert, Ragnar Ardal, Anders Claesson, Tomas Magnusson, Jay Pantone, Murray Tannock and Henning Ulfarsson.)

MSC2000: 05-04, 05A05, 05A15

"It goes as follows. Have a (as of now, human) mathematician get a brilliant idea. Teach that idea to a computer, and let the computer 'do research' using that idea." - Doron Zeilberger.

Our goal is to create a virtual combinatorist that can enumerate permutation classes, which are sets of permutations closed downwards by containment. We follow the mentality set out by Zeilberger. The first step, therefore, is to have "a brilliant idea". We call these *proof strategies*. In our case, we are always working with a permutation class $\mathcal{C}$, which is a subset of all permutations. For some subset S of all permutations, a proof strategy is a disjoint set of subsets $S_1, S_2, \ldots, S_k$, such that when each subset is intersected with $\mathcal{C}$ their disjoint union is equal to the intersection of S and $\mathcal{C}$, i.e.

$$(S \cap \mathcal{C}) = (S_1 \cap \mathcal{C}) \sqcup (S_2 \cap \mathcal{C}) \sqcup \ldots \sqcup (S_k \cap \mathcal{C}).$$

Consider the tree where the root node is some permutation class $\mathcal{C}$. The children of a node are subsets obtained by some proof strategy, thus form a disjoint union for their parent while working inside $\mathcal{C}$. If all the leaves of the tree are subsets of $\mathcal{C}$ then their disjoint union define a description for $\mathcal{C}$, while the tree itself is a proof certificate of this description. We call such trees *proof trees.*

This is where the virtual combinatorist comes in. We train it how to use proof strategies and have it search for a proof tree. Our virtual combinatorist has been able to prove the enumeration of some permutation classes which have had entire papers devoted to them. After the virtual combinatorist has "researched" for a bit, some permutation classes may still have been beyond the strategies, and so the cycle starts again, the mathematicians again try to find new proof strategies.

[1] D. Zeilberger. Enumeration schemes and, more importantly, their automatic generation. *Ann. Comb.*, 2(2):185195, 1998.

Some results on the roots of the independence polynomial of graphs

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MSC2000: 05C31

The independence polynomial of a graph G is

$$I(G,x)=\sum_{k\geq 0} i_k(G)x^k,$$

where $i_k(G)$ denotes the number of independent sets of G of size k (note that $i_0(G)=1$). In this talk we will investigate some properties of this graph polynomial.

In particular we will give a new proof for real-rootedness of the independence polynomials of certain families of trees, which includes centipedes (Zhu's theorem, see [2]), caterpillars (Wang and Zhu's theorem, see [3]), and we will prove a conjecture of Galvin and Hilyard about the real-rootedness of the independence polynomial of the Fibonacci trees (Conj. 6.1. of [1]). Moreover we will see that the root counting measure of the independence polynomial of any sequence of d-regular graphs with girth tending to infinity weakly converges to a measure on the reals.

[1] D. Galvin and J. Hilyard. The independent set sequence of some families of trees, *arXiv preprint* arxiv:1701.02204. 2017.

[2] Z. F. Zhu. The unimodality of independence polynomials of some graphs. *Australasian Journal of Combinatorics*, 38:27–33, 2007.

[3] Y. Wang and B. X. Zhu. On the unimodality of independence polynomials of some graphs. *European Journal of Combinatorics*, 32(1):10–20, 2011.

Tuesday 4.40 Auditorium C

Prolific permutations and permuted packings

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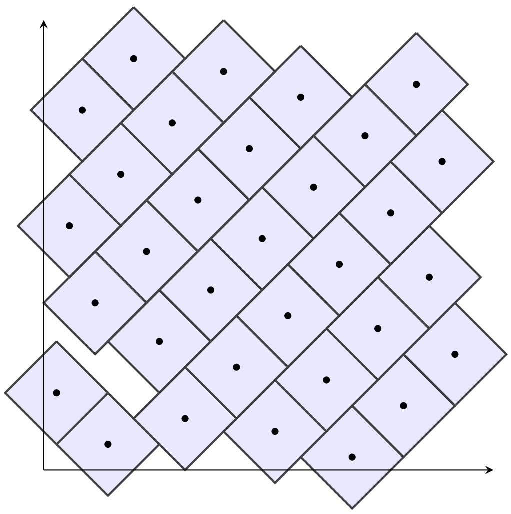
(This talk is based on joint work with Cheyne Homberger and Bridget Tenner.)

MSC2000: 05A05, 05B40, 06A07

A permutation of length n is *k-prolific* if each of the $(n-k)$-subsets of the entries in its one-line notation forms a distinct subpermutation. We give a complete characterization of k-prolific permutations for each k, and present an outline of the proof that k-prolific permutations of length n exist for every $n \geqslant k^2/2 + 2k + 1$, and that none exist of smaller size. Key to these results is a natural bijection between k-prolific permutations and certain *permuted packings* of diamonds.



The permuted diamond packing corresponding to a 6-prolific permutation.

Non-overlapping codes

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MSC2000: 94B60

We say that words u and v are *overlapping* if a non-empty proper prefix of u is equal to a non-empty proper suffix of v, or if a non-empty proper prefix of v is equal to a non-empty proper suffix of u. So, for example, the binary words 00000 and 01111 are overlapping; so are the words 10001 and 11110. However, the words 11111 and 01110 are non-overlapping. A q-ary length n code is non-overlapping if all codewords u and v (not necessarily distinct) are non-overlapping. The basic question is: what is the largest number $C(n,q)$ of codewords of a q-ary non-overlapping code of length n?

Non-overlapping codes were introduced by V.I. Levenshtein in 1964 (under the name 'strongly regular code'; in later papers he refers to 'codes without overlaps'). There has been recent interest in these codes after they were independently rediscovered by Bajić and Stojanović. Non-overlapping codes are interesting for synchronisation applications: they are comma-free codes where errors do not propagate indefinitely.

This talk surveys some upper and lower bounds for $C(n,q)$ (the best of which are surprisingly close), and describes a tantalising open problem for binary codes.

Tight lower bounds for the complexity of multicoloring

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(This talk is based on joint work with Łukasz Kowalik, Michał Pilipczuk, Arkadiusz Socała and Marcin Wrochna.)

MSC2000: 05C15, 05C85

In the *multicoloring* problem, also known as $(a : b)$*-coloring* or b*-fold coloring*, we are given a graph G and a set of a colors, and the task is to assign a subset of b colors to each vertex of G so that adjacent vertices receive disjoint color subsets. This natural generalization of the classic coloring problem (the $b = 1$ case) is equivalent to finding a homomorphism to the Kneser graph $KG_{a,b}$. It is tightly connected with the fractional chromatic number, and has multiple applications within computer science.

We study the complexity of determining whether a graph has an $(a : b)$-coloring. As shown by Cygan et al. [1], given an arbitrary n-vertex graph G and h-vertex graph H one cannot determine in time $2^{o(\log h)\cdot n}$ whether G admits a homomorphism to H, unless the Exponential Time Hypothesis (ETH) fails. Despite the fact that when H is the Kneser graph $KG_{a,b}$ we have $h = \binom{a}{b}$, Nederlof [3] showed a $(b+1)^n \cdot \mathrm{poly}(n)$-time algorithm for $(a : b)$-coloring. Our main result is that this is essentially optimal: there is no algorithm with running time $2^{o(\log b)\cdot n}$ unless the ETH fails. The crucial ingredient in our hardness reduction is the usage of *detecting matrices* of Lindström [2], which is a combinatorial tool that, to the best of our knowledge, has not yet been used for proving complexity lower bounds.

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[2] B. Lindström. On a combinatorial problem in number theory. *Canadian Mathematical Bulletin*, 8(4):477–490, 1965.

[3] J. Nederlof. Inclusion exclusion for hard problems. *Master's thesis, Utrecht University*, 2008.

Plenary lecture

Tuesday 9.00 Main Auditorium

Large-scale structures in random graphs

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MSC2000: 05C80

In recent years there has been much progress in graph theory on questions of the following type. What is the threshold for a certain large substructure to appear in a random graph? When does a random graph contain all structures from a given family? And when does it contain them so robustly that even an adversary who is allowed to perturb the graph cannot destroy all of them? I will survey this progress, and highlight the vital role played by some newly developed methods, such as the sparse regularity method, the absorbing method, and the container method. I will also mention many open questions that remain in this area.

Recent Results on Chromatic Polynomials

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(This talk is based on joint work with A. Erey, D. Wagner.)

MSC2000: 05C15

The chromatic polynomial of a finite graph G counts the number of ways the vertices of a graph can be coloured with x colours so that adjacent vertices receive different colours. Chromatic polynomials were initially introduced while working on the Four Colour Conjecture, and have been studied not only for what they can say about chromatic theory but also as analytic and algebraic objects of interest in their own right. In this talk I will present recent work on new bounds for chromatic polynomials as well as results on the real part, imaginary part and moduli of their roots.

Colourings of group divisible designs

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(This talk is based on joint work with Peter Danziger, Jeff Dinitz, Diane Donovan and David Pike.)

MSC2000: 05B05, 05C15

An *m-colouring* of a design is an assignment of m colours to the points of the design so that in each block, there are at least two vertices of different colours. Usually, we are interested in colouring a design using the smallest number of colours possible, and define this number to be the design's *chromatic number*.

In a *group divisible design*, the point set V is partitioned into subsets called groups, and the blocks of the design are subsets of V such that each pair of points occurs together in a group or in exactly one block, but not both. In this talk, we present recent progress on colourings of group divisible designs.

Unimodal inversion sequences and pattern-avoiding classes

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(This talk is based on joint work with Walter Stromquist.)

MSC2000: 05A05

Mansour and Shattuck [4] have shown that exactly 9 classes of permutations avoiding triples of patterns of length 4 have the enumeration sequence that is the binomial transform of Fine's sequence [5]. Four of these classes were separately shown previously to be enumerated by this sequence (see [1, 2, 3, 7]), while five others are new. The same sequence also enumerates unimodal inversion sequences (inversion sequences are reversals of Lehmer codes) [6].

We generalize these findings in two different ways.

- For some of the above sets of patterns T_i $(i = 1, \dots, 9)$, we find encodings of all permutations by inversion sequences so that the inversion sequences corresponding to the set $\mathrm{Av}(T_i)$ of T_i-avoiders are unimodal.
- For the same sets of patterns T_i, we use the above encodings of $\mathrm{Av}(T_i)$ to generalize the Wilf-equivalence of T_i's to a Wilf-equivalence of families of similarly related sets of patterns of any size, obtained by inflating a certain entry of each permutation in T_i by the same block. We also conjecture similar generalizations for all but one of the remaining classes.

[1] M. Albert, R. Aldred, M. Atkinson, C Handley, D. Holton, D. McCaughan, H. van Ditmarsch, Sorting Classes, *Electron. J. Combin.* **12** (2005), R31.

[2] J. Bloom, A. Burstein, Egge triples and unbalanced Wilf-equivalence, *Austral. J. Combin.* **64(1)** (2016), 232-251.

[3] R. Brignall, S. Huczyncka, V. Vatter, Simple permutations and algebraic generating functions, *J. Combin. Th. Ser. A*, **115** (2008), 423-441.

[4] T. Mansour, M. Shattuck, Nine classes of permutations enumerated by binomial transform of Fine's sequence, *J. Discrete Appl. Math.*, in press, https://doi.org/10.1016/j.dam.2017.04.015.

[5] OEIS Foundation Inc. (2017), The On-Line Encyclopedia of Integer Sequences, http://oeis.org/A033321.

[6] C.D. Savage, Inversion sequences, 12th International Conference on Permutation Patterns (PP 2014), Johnson City, TN, USA, July 7-11, 2014.

[7] R. Smith, V. Vatter, A stack and a pop stack in series, *Austral. J. Combin.* **58(1)** (2014), 157-171.

Synchronization, association schemes and Steiner systems

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MSC2000: 05B05

The notion of synchronization from automata theory has been extended to permutation groups [1]. I extend it further, to association schemes. An important class of association schemes introduced in the context of coding theory by Delsarte [2] consists of the *Johnson schemes*. Investigating synchronization and the related property of separation for these schemes leads to a conjecture which would extend the result of Keevash [3] on the existence of Steiner systems. I will mention evidence for this conjecture, some of it joint work with John Bamberg.

[1] João Araújo, Peter J. Cameron and Benjamin Steinberg, Between primitive and 2-transitive: Synchronization and its friends, arXiv:1511.03184

[2] Philippe Delsarte, An algebraic approach to the association schemes of coding theory, *Philips Res. Repts. Supplement* **10** (1973).

[3] Peter Keevash, The existence of designs, arXiv:1401.3665

Coloring graphs from random lists

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MSC2000: 05C15

The topic of this talk is list coloring of graphs. In this model each vertex of a graph is assigned a list (set) of colors and the task is then to construct a proper coloring of the graph where each vertex gets a color from its list. I shall discuss a relatively new variation on list coloring where the color lists are assigned randomly: let $G = G(n)$ be a graph on n vertices, and assign to each vertex v of G a list $L(v)$ of colors by choosing each list independently and uniformly at random from all k-subsets of a color set of size $\sigma(n)$. I will discuss various conditions implying that with probability tending to 1, as $n \to \infty$, G has a proper coloring with colors from the random lists.

Plenary lecture

Friday 9.00 Main Auditorium

The spt-Function of Andrews

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MSC2000: 05A17, 11P83, 11F03, 11F33, 05A20

The spt-function $\mathrm{spt}(n)$ was introduced by Andrews as the weighted counting of partitions of n with respect to the number of occurrences of the smallest part. Andrews showed that $\mathrm{spt}(5n+4) \equiv 0 \pmod 5, \mathrm{spt}(7n+5) \equiv 0 \pmod 7$ and $\mathrm{spt}(13n+6) \equiv 0 \pmod{13}$. Since then, congruences of $\mathrm{spt}(n)$ have been extensively studied. Folsom and Ono obtained congruences of $\mathrm{spt}(n)$ mod 2 and 3. They also showed that the generating function of $\mathrm{spt}(n)$ mod 3 is related to a weight 3/2 Hecke eigenform with Nebentypus. Combinatorial interpretations of congruences of $\mathrm{spt}(n)$ mod 5 and 7 have been found by Andrews, Garvan and Liang by introducing the spt-crank of a vector partition. Chen, Ji and Zang showed that the set of partitions counted by $\mathrm{spt}(5n+4)$ (or $\mathrm{spt}(7n+5)$) can be divided into five (or seven) equinumerous classes according to the spt-crank of a doubly marked partition. Let $N_S(m,n)$ denote the net number of S-partitions of n with spt-crank m. Andrews, Dyson and Rhoades conjectured that $\{N_S(m,n)\}_m$ is unimodal for any n. Chen, Ji and Zang gave a constructive proof of this conjecture.

In this survey, we summarize developments on congruence properties of $\mathrm{spt}(n)$ established by Andrews, Bringmann, Folsom, Garvan, Lovejoy and Ono et al., as well as their combinatorial interpretations. Generalizations and variations of the spt-function are also discussed. Moreover, we give an overview of asymptotic formulas of $\mathrm{spt}(n)$ obtained by Ahlgren, Andersen and Rhoades et al. We conclude with some conjectures on inequalities on $\mathrm{spt}(n)$, which are reminiscent of inequalities on the partition function $p(n)$ due to DeSalvo and Pak, and Bessenrodt and Ono. Finally, we observe that, beyond the log-concavity, $p(n)$ and $\mathrm{spt}(n)$ satisfy higher order inequalities based on polynomials arising in the invariant theory of binary forms. In particular, we conjecture that the higher order Turán inequality $4(a_n^2 - a_{n-1}a_{n+1})(a_{n+1}^2 - a_n a_{n+2}) - (a_n a_{n+1} - a_{n-1}a_{n+2})^2 > 0$ holds for $p(n)$ when $n \geq 95$ and for $\mathrm{spt}(n)$ when $n \geq 108$.

Plenary lecture

Thursday 11.15 Main Auditorium

New applications of Riordan arrays

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MSC2000: 05A15

Let $\kappa[[z]]$ be the ring of formal power series over an integral domain κ. A *Riordan array* denoted (g, f) is an infinite lower triangular matrix constructed out of two functions $g, f \in \kappa[[z]]$ with $f(0) = 0$ in such a way that its kth column generating function is gf^k for $k \geq 0$. In many contexts we see that the Riordan arrays are used as a machine to generate new approaches in combinatorics.

This talk is devoted to discussing new applications of Riordan arrays. We introduce some applications arising in combinatorics, graph theory, and the Riemann hypothesis, respectively. More specifically, the talk consists of three parts as follows. First, we introduce the concept of q-analogue for Riordan arrays (g, f) and its applications to combinatorics. In particular, we show that the q-Laguerre polynomials defined by the Eulerian generating function can be expressed not only in terms of the inversions of a permutation but also in terms of q-rook numbers. Second, we introduce the concept of Riordan graphs associated to Riordan arrays (g, f) modulo 2 over the ring of integers. We denote the graph by $RG_n(g, f)$ and we explore their structural properties. Finally, we use the Riordan arrays (g, f) to find a large class of matrices called the Riordan-Redheffer matrix of order n with the same determinant as the Mertens function $M(n) = \sum_{k=1}^{n} \mu(k)$ where μ is the Möbius function. It is well-known that the Riemann hypothesis is true if and only if $M(n) = O(n^{1/2+\epsilon})$ for all positive ϵ. We give several examples of Riordan-Redheffer matrix that reveal interesting spectral properties, and some conjectures on its eigenvalues will be posed.

[1] W. W. Barrett and T. J. Jarvis, Spectral properties of a matrix of Redheffer, *Linear Algebra Appl.*, 162-164: 673-683, 1992.

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On mark sequences in bipartite multidigraphs

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MSC2000: 05C20

A tournament is an irreflexive, complete, asymmetric digraph, and the score s_v of a vertex v in a tournament is the number of arcs directed away from that vertex. A sequence $S = [s_1, s_2, \cdots, s_n]$ of non-negative integers in non-decreasing order is a score sequence if it realizes some tournament. Landau in 1953 characterized the score sequences of a tournament. An oriented graph is a digraph with no symmetric pairs of directed arcs and without self loops. In 1991, Avery obtained the characterization of score sequences in oriented graphs. An r-digraph D is an orientation of a multigraph that is without loops and contains at most r edges between any pair of distinct vertices. The mark of a vertex v_i in D is $p_i = r(n-1) + d_i^+ - d_i^-$, where d_i^+ and d_i^- are respectively the outdegree and indegree of v_i. The sequence of marks is called the mark sequence of D. One representation of a multidigraph is a competition in which n participants play each other r times and the result includes the ties. Pirzada and Samee extended the concept of scores to digraphs. We characterize mark sequences in bipartite multidigraphs, which also result in construction algorithms.

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Difference covering arrays delivering high rate LDPC codes

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(This talk is based on joint work with Asha Rao and Emine Yazici.)

MSC2000: 05B99, 05B15, 62K99

In this talk I will presents a combinatorial construction of low-density parity-check (LDPC) codes from difference covering arrays. Gallagher originally constructed LDPC by randomly allocating bits in a sparse paritycheck matrix. However in the past 20 years researchers have used a variety of more structured approaches, with recent constructions using balanced incomplete block designs (BIBDs) and Latin squares over finite fields delivering well-structured LDPC. However many of these constructions have some limitations. Here I present a construction of LDPC codes of length $4n^2 - 2n$ for all n using the cyclic group of order $2n$. These codes achieve high information rate, have girth at least 6 and have minimum distance 6 for n odd.

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Path-factors and odd components

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(This talk is based on joint work with M. Furuya and K. Ozeki.)

MSC2000: 05C70

For a graph G and an integer $l \geq 1$, we let $c_l(G)$ denote the number of components of G having order l. For an integer $l \geq 2$, we let P_l denote the path of order l. By a $\{P_2, P_{2k+1}\}$-factor of a graph G, we mean a spanning subgraph of G each of whose components is isomorphic to P_2 or P_{2k+1}.
It is well-known that a graph G has a $\{P_2, P_3\}$-factor if and only if $c_1(G-X) \leq 2|X|$ for all $X \subseteq V(G)$. In [1], it was shown that if a graph G satisfies $c_1(G-X) + \frac{2c_3(G-X)}{3} \leq \frac{4|X|}{3}$ for all $X \subseteq V(G)$, then G has a $\{P_2, P_5\}$-factor. The following conjecture was also made in [2].

Conjecture 1. *Let $k \geq 3$ be an integer, and let G be a graph such that $\sum_{0 \leq j \leq k-1} c_{2j+1}(G-X) \leq \frac{(4k+6)|X|}{8k+3}$ for all $X \subseteq V(G)$. Then G has a $\{P_2, P_{2k+1}\}$-factor.*

For $k = 3, 4$, Conjecture 1 was affirmatively settled in [3]. For $k \geq 5$, we have recently proved the following theorem.

Theorem 2. *Let $k \geq 5$, be an integer, and let G be a graph such that $\sum_{0 \leq j \leq k-1} c_{2j+1}(G-X) \leq \frac{5|X|}{6k^2}$ for all $X \subseteq V(G)$. Then G has a $\{P_2, P_{2k+1}\}$-factor.*

On the other hand, for $k \geq 29$, we have constructed examples of graphs G with no $\{P_2, P_{2k+1}\}$-factor such that $\sum_{0 \leq j \leq k-1} c_{2j+1}(G-X) \leq \frac{(32k+141)|X|}{72k-78}$ for all $X \subseteq V(G)$, which show that Conjecture 1 is false for $k \geq 36$. These examples are related to examples of graphs constructed in [1] in connection with the famous conjecture of Chvátal concerning the hamiltonicity of graphs with high toughness.
In this talk, I will describe how we have modified the idea used in [1] to construct our examples. I will also describe the possibility of improving the coefficient $\frac{5}{6k^2}$ in Theorem 2.

[1] D. Bauer, H. J. Broersma, and H. J. Veldman. Not every 2-tough graph is Hamiltonian. *Discrete Applied Mathematics*, 99:317–321, 2000.

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[3] Y. Egawa and M. Furuya, Path-factors involving paths of order seven and nine. *Theory and Applications of Graphs*, 3(1):#5, 2016.

The structure of the consecutive pattern poset

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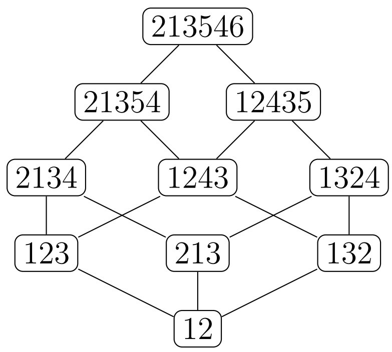
(This talk is based on joint work with Peter McNamara.)

MSC2000: 06A07 (Primary), 05A05, 05A16, 05E45, 52B22, 60C05 (Secondary)

The consecutive pattern poset $\mathcal{P}$ is the infinite partially ordered set of all permutations where $\sigma \leq \tau$ if τ has a subsequence of adjacent entries in the same relative order as the entries of σ. For example, the interval $[12, 213546]$ in $\mathcal{P}$ in shown here:



A recursive expression for the Möbius function of $\mathcal{P}$ was given by Bernini–Ferrari–Steingrímsson [1] and Sagan–Willenbring [3], inspired by a (still open) question of Wilf [4] asking for the Möbius function of intervals in the poset defined by classical pattern containment, where the entries in a subsequence are not required to be adjacent.

Following up on their work, we study the structure of the intervals in $\mathcal{P}$ from topological, poset-theoretic, and enumerative perspectives. In particular, we prove that all intervals are rank-unimodal and strongly Sperner, and we characterize disconnected and shellable intervals. We also show that most intervals are not shellable and have Möbius function equal to zero. Most of this talk is based on the results in [2].

[1] A. Bernini, L. Ferrari and E. Steingrímsson. The Möbius function of the consecutive pattern poset. *Electron. J. Combin.* 18, Paper 146, 2011.

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[3] B. Sagan and R. Willenbring. Discrete Morse theory and the consecutive pattern poset. *J. Algebraic Combin.* 36:501-514, 2012.

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Tuesday 10.30 Main Auditorium

Adversarial games on interval and permutation graph representations

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(This talk is based on joint work with Lorna Stewart.)

MSC2000: 05C62

I will describe a class of combinatorial games on graphs in which two players antagonistically build a geometric representation of a given graph. For a large class of these games, determining whether a given instance is a winning position for the next player is PSPACE-hard. I will outline this hardness result, and mention some cases in which identifying winning positions is efficiently computable. I will conclude by posing several open questions, and pondering aloud about the relationship between the hardness of solving these games and the number of canonical representations of an input graph.

Large Cayley graphs of diameter three

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(This talk is based on joint work with James Tuite.)

MSC2000: 05C25, 05C35

The degree-diameter problem seeks to find the largest possible number of vertices in a graph having given diameter and given maximum degree. Very often the problem is studied for restricted families of graph such as vertex-transitive or Cayley graphs, with the goal being to find an infinite family of graphs with good asymptotic properties.

A simple counting argument shows that for a fixed diameter k, an undirected graph of maximum degree d has order (asymptotically) at most d^k. Our goal is to find a family of graphs of diameter k such that as the degree d becomes large, the order of our graphs is within a multiplicative constant of the asymptotic upper bound.

Cayley graphs are a natural means to attack the degree-diameter problem, because the constraint on the diameter of the graph translates naturally into a statement about multiplication within the associated group. We are thus able to express our graph problem directly in group-theoretic terms.

Much of the existing literature is focused on the diameter two case, with a variety of families of group employed to yield good bounds for diameter two Cayley graphs. In this case and also for larger diameters, a common approach is to use some kind of semidirect product construction which can often yield useful groups which may be covered efficiently by small generating sets.

We will describe a new construction, which instead uses matrix groups over finite fields. This new construction has been used to improve the existing asymptotic bounds for Cayley graphs of diameter 3, valid for all sufficiently large degrees d.

Tutte Centenary Lecture

Wednesday 9.00 Main Auditorium

W. T. Tutte: Graph theory as he had grown it

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MSC2000: 01A60, 05-03, 05C31, 05C15, 00A08

This year is the centenary of the birth of W. T. (Bill) Tutte (1917–2002). This talk reviews some of his earliest work, which contains seeds from which much of modern graph theory has grown. We emphasise the Tutte polynomial [2, 3, 4, 5], which plays a central role in the study of counting on graphs.

Tutte was a chemistry undergraduate at Cambridge in the late 1930s when he and three friends became interested in the recreational puzzle of "squaring the square" [1]. The ideas they developed and used to solve this puzzle drew Tutte into the serious study of mathematics, especially graph theory. After an astonishingly successful detour into cryptanalysis during the Second World War, Tutte gained his PhD at Cambridge [3]. He then moved to Toronto and, later, Waterloo. Although he worked in a relatively young field, he became arguably one of the greatest mathematicians of the twentieth century.

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Automorphisms of the Gómez Graphs

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MSC2000: 05C25

In extremal graph theory we often pursue the goal of maximising chosen properties of a graph with respect to given constraints. One problem within the area is the degree diameter problem, in which we attempt to find the graphs of largest possible size with a given degree and diameter. In this talk we consider the best known infinite family of directed graphs, the Gómez Graphs, with large size for a given degree and diameter. Their construction leads trivially to the observation that the symmetric group is a subgroup of their full automorphism group. By showing that this obvious subgroup is in fact the full automorphism group we are able to show when this family of graphs is in fact Cayley.

On 1212-avoiding Restricted Growth Functions

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(This talk is based on joint work with Zhicong Lin.)

MSC2000: 05A05

Abstract

Restricted growth functions (RGFs) avoiding the pattern 1212 are in natural bijection with noncrossing partitions. Motivated by recent work of Campbell et al. [1], we study five classical statistics $\mathrm{bk}, \mathrm{ls}, \mathrm{lb}, \mathrm{rs}$ and rb on 1212-avoiding RGFs. We show the equidistribution of $(\mathrm{ls}, \mathrm{rb}, \mathrm{lb}, \mathrm{bk})$ and $(\mathrm{rb}, \mathrm{ls}, \mathrm{lb}, \mathrm{bk})$ on 1212-avoiding RGFs by constructing a simple involution. To our surprise, this result was already proved by Simion [2] 22 years ago via an involution on noncrossing partitions. Our involution, though turns out essentially the same as Simion's, is defined quite differently and has the advantage that makes the discussion more transparent. Consequently, a multiset-valued extension of Simion's result is discovered. Furthermore, similar approach enables us to prove the equidistribution of $(\mathrm{mak}, \mathrm{rb}, \mathrm{rs}, \mathrm{bk})$ and $(\mathrm{rb}, \mathrm{mak}, \mathrm{rs}, \mathrm{bk})$ on 1212-avoiding RGFs, where "mak" is a set partition statistic introduced by Steingrímsson [3].

Through two bijections to Motzkin paths, we also prove that the triple of classical permutation statistics $(\mathrm{exc}+1, \mathrm{den}, \mathrm{inv}-\mathrm{exc})$ on 321-avoiding permutations is equidistributed with the triple $(\mathrm{bk}, \mathrm{rb}, \mathrm{rs})$ on 1212-avoiding RGFs, which generalizes another result of Simion [2]. In the course, an interesting q-analog of the γ-positivity of Narayana polynomials is found.

Keywords: restricted growth function; pattern avoidance; noncrossing partitions; partition statistics; Narayana polynomials

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On lower bound for induced Ramsey numbers

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MSC2000: 05C55

We say that a graph F *strongly arrows* a pair of graphs (G,H) and write $F \xrightarrow{\text{ind}} (G,H)$ if any 2-colouring of its edges with red and blue leads to either a red G or a blue H appearing as induced subgraphs of F. *The induced Ramsey number*, $IR(G,H)$ is defined as $\min\{|V(F)| : F \xrightarrow{\text{ind}} (G,H)\}$. The existence of this number was proved independently by Deuber [1], Erdős, Hajnal and Pósa [2] and Rödl [6]. It is not much known about the behaviour of the induced Ramsey number and the results are mostly of asymptotic type. Moreover these results are generally upper bounds. The only known exact results (not concerning the pairs of small graphs) are for a pair of stars by Harary, Nešetril and Rödl [5], matching versus complete graphs by Gorgol and Łuczak [4] and for stars versus complete graphs by Gorgol [3]. Similarly as for lower bounds some general bound is known for triangle-free graphs versus complete graphs [3].

In the talk we will show the lower bound of the induced Ramsey number in terms of the independence and clique numbers. This bound is sharp, i.e. there are pairs of graphs for which this lower bound is actually the exact value of the induced Ramsey number.

[1] W. Deuber, *A generalization of Ramsey's theorem*, Infinite and finite sets (R. Rado A. Hajnal and V. Sós, eds.), vol. 10, North-Holland, pp. 323–332, 1975.

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[6] V. Rödl, *A generalization of Ramsey theorem*, Ph.D. thesis, Charles University, Prague, Czech Republic, pp. 211–220, 1973.

Some cyclic properties of graphs with a local Ore-type condition

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MSC2000: 05C45

A graph G is called an L_1-graph if, for each triple of vertices u, v, w with $d(u, v) = 2$ and $w \in N(u) \cap N(v)$, $d(u) + d(v) \geq |N(u) \cup N(v) \cup N(w)| - 1$. Asratian et al. [1] proved that connected L_1-graphs of order at least 3 such that $|N(u) \cap N(v)| \geq 2$ for every pair of vertices u, v with $d(u, v) = 2$ are Hamiltonian (with some exceptions).

In this talk we demonstrate that any nonhamiltonian cycle in such a graph can be extended to a larger cycle containing all vertices of the original cycle and at most two other vertices. Analogous results are given for paths in which the endpoints do not have any common neighbors. These results are sharp; in particular not every graph from this class is pancyclic.

[1] A. S. Asratian, H. J. Broersma, J. van den Heuvel, and H. J. Veldman. On graphs satisfying a local Ore-type condition. *J. Graph Theory*, 21(1):1–10, 1996.

Asymptotic enumeration by Khintchine-Meinardus probabilistic method: Necessary and sufficient conditions for exponential growth

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MSC2000: 05A15, 05A16, 05A17

The present paper (see ArXiv, 2016) is devoted to asymptotic enumeration of decomposable combinatorial structures, such as for example, weighted partitions. We prove the necessity of the main sufficient condition of Meinardus for subexponential rate of growth of the number of structures, having multiplicative generating functions of a general form and establish a new necessary and sufficient condition for the normal local limit theorem for the aforementioned structures. The latter result allows to encompass in our study structures with weights having gaps in their supports. The paper continues the work in [1] and [2].

[1] Granovsky, B, Stark, D . Developments in the Khintchine-Meinardus probabilistic method for asymptotic enumeration *EJC*, 22,4, 2015.

[2] Granovsky, B. and Stark, D. A Meinardus theorem with multiple singularities. *Comm. Math. Phys.* **314** 329–350, (2012).

Pentagonal Geometries

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(This talk is based on joint work with Klara Stokes, University of Skövde, Sweden & Tony Forbes, The Open University.)

MSC2000: 05B25, 51E12

The concept of a generalized polygon was introduced by Tits in 1959. Perhaps the simplest definition is the following. A *partial linear space* S is an ordered pair $(P, \mathcal{L})$ where P is a set of *points* and $\mathcal{L}$ is a set of *lines* such that every pair of points is incident with at most one line. Consider the bipartite point-line incidence graph G of S. If the girth of G is twice its diameter, then S is a *generalized polygon*. Clearly the incidence graph of an n-gon is the cyclic graph on $2n$ vertices.

By the Feit-Higman Theorem (1964), the only finite examples are *thin* (two points on each line or two lines on each point) or the diameter is 3, 4, 6 or 8. So in particular there are no generalized pentagons. In this talk I will present an alternative way to generalize the pentagon introduced by Simeon Ball et alia in [1] and discuss what is know about these structures.

[1] S. Ball, J. Bamberg, A. Devillers and K. Stokes. An alternative way to generalize the pentagon. *J. Combin. Des.*, 21:163–179, 2013.

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Bounds on traceability schemes

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(This talk is based on joint work with Ying Miao.)

MSC2000: 05D05, 94A60

The Stinson-Wei traceability scheme (known as traceability scheme) [5] was proposed for broadcast encryption as a generalization of the Chor-Fiat-Naor traceability scheme (known as traceability code) [1]. Cover-free family was introduced by Kautz and Singleton [4] in the context of binary superimposed code. Let $(\mathcal{X}, \mathcal{B})$ be a set system with $\mathcal{B} \subseteq \binom{\mathcal{X}}{w}$, $|\mathcal{X}| = v$, and $|\mathcal{B}| = M$.

(1) $(\mathcal{X}, \mathcal{B})$ is a *t-traceability scheme* t-TS(w, M, v) provided that for every choice of $s \leq t$ blocks $B_1, B_2, \ldots, B_s \in \mathcal{B}$ and for any w-subset $T \subseteq \bigcup_{1 \leq j \leq s} B_j$, there does not exist a block $B \in \mathcal{B} \setminus \{B_1, B_2, \ldots, B_s\}$ such that $|T \cap B| \geq |T \cap B_j|$ for all $1 \leq j \leq s$.

(2) $(\mathcal{X}, \mathcal{B})$ is a *t-cover-free family* t-CFF(w, M, v) provided that for any $t+1$ distinct blocks $B_0, B_1, \ldots, B_t \in \mathcal{B}$, we have

$$B_0 \nsubseteq \bigcup_{1 \leq i \leq t} B_i.$$

In this talk, first, we find a new relationship between a TS and a CFF, that is, a t-TS is a t^2-CFF. Based on this interesting discovery and some known results of CFF by Erdős, Frankl and Füredi [3], we derive new upper bounds for the number of blocks in a t-TS, which improve the best known bounds in [2]. Next, by using combinatorial designs, we construct several infinite families of optimal t-TS which attain our new upper bounds. We also provide a constructive lower bound for t-TS, the size of which has the same order of magnitude as our general upper bound.

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[5] D. R. Stinson and R. Wei. Combinatorial properties and constructions of traceability schemes and frameproof codes. *SIAM J. Discrete Math.*, 11:41–53, 1998.

Independent sets in hypergraphs and Ramsey properties of graphs and the integers

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(This talk is based on joint work with Katherine Staden and Andrew Treglown.)

MSC2000: 05C55, 05C60, 05D10, 05C65, 05C80

Many important problems in combinatorics and other related areas can be phrased in the language of independent sets in hypergraphs. Recently Balogh, Morris and Samotij [1], and independently Saxton and Thomason [8] developed very general container theorems for independent sets in hypergraphs; both of which have seen numerous applications to a wide range of problems. We use the container method to prove results that correspond to problems concerning tuples of disjoint independent sets in hypergraphs.

We generalise the random Ramsey theorem of Rödl and Ruciński [5, 6, 7] by providing a resilience analogue. This result also implies the random version of Turán's theorem due to Conlon and Gowers [2] and Schacht [9]. We prove a general subcase of the asymmetric random Ramsey conjecture of Kohayakawa and Kreuter [4]. Both of the above results in fact hold for uniform hypergraphs. We also strengthen the random Rado theorem of Friedgut, Rödl and Schacht [3] by proving a resilience version of the result.

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[4] Y. Kohayakawa and B. Kreuter, Threshold functions for asymmetric Ramsey properties involving cycles, *Random Structures & Algorithms* **11** (1997), 245–276.

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[6] V. Rödl and A. Ruciński, Random graphs with monochromatic triangles in every edge coloring, *Random Structures & Algorithms* **5** (1994), 253–270.

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Domination and Independence of Bishop Graphs

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(This talk is based on joint work with Dr Stephanie Perkins, Dr Siân Jones and Prof. Paul Roach.)

MSC2000: 05C69, 00A08

For a given chess piece and a given board a graph G can be constructed. In this graph each vertex corresponds to a square of the board and an edge exists between two vertices A and B if a chess piece on the square represented by A can reach the square represented by B in a single move. The vertex independence number $\beta_0(G)$ and domination number $\gamma(G)$ can be determined for this chess graph by studying corresponding positional chess problems. This talk will identify these positional chess problems and explore the solutions for the bishop chess piece on a variety of different boards, including abstractions to hexagonal chess boards.

Some Multicolor Bipartite Ramsey Numbers Involving Cycles And A Small Number Of Colors

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(This talk is based on joint work with Ernst J. Joubert.)

MSC2000: 05C55

For bipartite graphs $G_1, G_2, \ldots, G_k$, the bipartite Ramsey number $b(G_1, G_2, \ldots, G_k)$ is the least positive integer b so that any coloring of the edges of $K_{b,b}$ with k colors will result in a copy of G_i in the ith color for some i. In this paper, our main focus will be to bound the following numbers: $b(C_{2t_1}, C_{2t_2})$ and $b(C_{2t_1}, C_{2t_2}, C_{2t_3})$ for all $t_i \geq 3$, $b(C_{2t_1}, C_{2t_2}, C_{2t_3}, C_{2t_4})$ for $3 \leq t_i \leq 9$, and $b(C_{2t_1}, C_{2t_2}, C_{2t_3}, C_{2t_4}, C_{2t_5})$ for $3 \leq t_i \leq 5$. Furthermore, we will also show that these mentioned bounds are generally better than the bounds obtained by using the best known Zarankiewicz-type result.

A variation of Ryser's theorem for partial $(\nu_1, \cdots, \nu_n)$-latinized squares

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MSC2000: 05B15

In 1951 Ryser gave a necessary and sufficient condition for an $r \times s$ partial latin rectangle b be extended to a $p \times p$ latin square P. Hilton and Johnson later showed that Ryser's condition is equivalent to the requirement that Hall's condition be satisfied (more particularly, that one of the main inequalities constituting Hall's condition be satisfied). We consider here the following variation of Ryser's theorem. We have n symbols, $\sigma_1, \cdots, \sigma_n$ where $n \geq p$, and n positive integers $\nu_1, \nu_2, \cdots, \nu_n$ with $1 \leq \nu_i \leq p$ $(1 \leq i \leq n)$ and $\sum_{i=1}^{n} = p^2$. An $r \times s$ partial $(\nu_1, \cdots, \nu_n)$-latinized rectangle R is an $r \times s$ matrix with cells filled from $\{\sigma_1, \cdots, \sigma_n\}$ in such a way that each symbol σ_i occurs at most once in any row and at most once in any column, and at most ν_1 times altogether. If $r = s = p$ then we have a $(\nu_1, \cdots, \nu_n)$-latinized square, and if also $n = p$ we have a latin square of order n. It is known that a $(\nu_1, \cdots, \nu_n)$-latinized square exists for every choice of $\nu_1, \cdots, \nu_n$ with $\sum_{i=1}^{n} \nu_i = p^2$ and $1 \leq \nu_i \leq p$. We show that a partial $r \times s$ $(\nu_1, \cdots, \nu_n)$-latinized rectangle can be completed to a $p \times p$ latinized square if and only if P satisfies Hall's $(\nu_1, \cdots, \nu_n)$ Constrained Condition, where P is the $p \times p$ matrix with R in the top left-hand corner, the other cells being blank, and Hall's $(\nu_1, \cdots, \nu_n)$-constrained condition is a suitable generalization of Hall's condition for a System of District Representations.

Tilings in graphons

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(This talk is based on joint work with Martin Doležal, Ping Hu, and Diana Piguet.)

MSC2000: 05C35, 05C80

The talk is based on papers available as [1, 2, 3].

We introduce a counterpart to the notion of vertex disjoint tilings by copy of a fixed graph F to the setting of graphons. The case $F = K_2$ gives the notion of matchings in graphons. We give a transference statement that allows us to switch between the finite and limit notion, and derive several favorable properties, including the LP-duality counterpart to the classical relation between the fractional vertex covers and fractional matchings/tilings, and discuss connections with property testing. We also study the structure of the matching-, and vertex cover- *polytons* which are extensions of the respective polytope concepts to the setting of graphons.

We give several applications of our theory to finite graphs. The most notable of these is a strengthening of a theorem of Komlós [4] in which he determined the minimum degree threshold for tiling a given fraction of a host graph by copies of an arbitrary graph F.

[1] M. Doležal and J. Hladký. Matching polytons. arXiv: 1606.06958.

[2] J. Hladký, P. Hu, and D. Piguet. Tilings in graphons. arXiv: 1606.03113.

[3] J. Hladký, P. Hu, and D. Piguet. Komlós's tiling theorem via graphon covers. arXiv: 1607.08415.

[4] J. Komlós, Tiling Turán Theorems, *Combinatorica* 20(2):203–218, 2000.

Plenary lecture

Monday 9.00 Main Auditorium

Switching techniques for edge decompositions of graphs

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MSC2000: 05C70, 05B07, 05C65

This talk concerns a class of techniques, sometimes referred to as *edge switching techniques*, that enable a new edge decomposition of a graph to be obtained from an existing one by interchanging edges between the subgraphs in the decomposition. These techniques can be viewed as generalisations of classical path switching methods for proper edge colourings. Their use in other edge decomposition settings dates back at least to 1980, but the last ten years have seen them rapidly developed and employed to resolve Lindner's conjecture on embedding partial Steiner triple systems, Alspach's cycle decomposition problem, and numerous other questions. I aim to give a gentle introduction to these techniques and to some of their most significant applications beyond edge colouring.

Graph classes under homomorphic image order

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University of St Andrews

(This talk is based on joint work with Nik Ruskuc.)

MSC2000: 05C60

Combinatorial structures have been considered under various different orders, including substructure order (both standard and induced forms), minor order and homomorphism order. In this talk, I will introduce and discuss the homomorphic image order, corresponding to the existence of a surjective homomorphism between two structures. I will focus on partial well-order and antichains, exploring how the homomorphic image order behaves in the context of graphs and graph-like structures. In particular, I will discuss a near-complete characterization of partially well-ordered avoidance classes with one obstruction.

Subgraph counts for dense graphs with specified degrees

Mikhail Isaev

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School of Mathematics and Statistics, UNSW Sydney

(This talk is based on joint work with C. Greenhill and B.D. McKay.)

MSC2000: 05C80, 05A16

We consider a uniformly chosen random graph G with given degree sequence in the dense range (degrees approximately a constant fraction of the number of vertices). For a given graph H, we find expected numbers of subgraphs and induced subgraphs of G isomorphic to H. Based on results of [2], these problems are reduced to determining of the expectation of certain functions of a random permutation. This is done by applying a general theory (developed in [1]) for the exponential of a martingale. As illustrations, we present formulas for expected numbers of perfect matchings, cycles and spanning trees in this random graph model.

[1] M. Isaev and B.D. McKay. Complex martingales and asymptotic enumeration. *Random Structures and Algorithms*(to appear), e-preprint arXiv:1604.08305.

[2] B.D. McKay. Subgraphs of dense random graphs with specified degrees. *Combinatorics, Probability and Computing*, 20:413–433, 2011.

Richard Rado Lecture

Friday 11.15 Main Auditorium

Ramsey-Type Problems in Permutations

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Computer Science Institute, Charles University in Prague

MSC2000: 05C05, 05D10

In my talk, I will survey several Ramsey-theoretic notions related to the combinatorics of permutations. I will focus on two main areas.

First, I will talk about the structural Ramsey properties of hereditary classes of permutations. I will introduce several structural properties of hereditary permutation classes that are inspired by Ramsey theory, and I will explain the relationships between them. I will then show how some of these structural properties can be helpful when dealing with enumeration problems on permutation classes.

In the second part of my talk, I will look at more quantitative questions; specifically, I will survey the known estimates for the Ramsey numbers of permutation matrices, and the closely related results on Ramsey numbers of sparse matrices and sparse ordered graphs.

Ramsey Problems for Odd Cycles

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Queen Mary, University of London

(This talk is based on joint work with A. Nicholas Day.)

MSC2000: 05D10

It is easy to see that every k-colouring of the edges of the complete graph on $2^k + 1$ vertices contains a monochromatic odd cycle. Erdős and Graham asked what can be said about the *shortest* monochromatic odd cycle which is guaranteed in such a colouring. We show that this is unbounded in the sense that for any positive integer r there exists an integer k and a k-colouring of K_{2^k+1} with no monochromatic odd cycle of length less than r.

We use these colourings to give new lower bounds on the k-colour Ramsey number of the odd cycle, disproving a conjecture of Bondy and Erdős.

(1,1,2,3)-Colourings of Subcubic Graphs

Prof E. Jonck

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University of the Witswatersrand

(This talk is based on joint work with Prof P. Dankelmann, Mr R.J. Maartens and Mr O. Nkuna.)

MSC2000: 05C10

For $1 \leq i \leq k$, let $S = (x_1, x_2, ..., x_k)$ be a non-decreasing sequence of integers such that $x_i \in \mathbb{Z}^+$. An S-packing colouring of a graph G is a function $\rho : V(G) \longmapsto \{x_1, x_2, ..., x_k\}$ such that for any two vertices $u, v \in V(G)$, $\rho(u) = \rho(v) = x_i$ if and only if the distance between u and v is greater than x_i. [1] asked whether it is true that any subcubic graph except the Petersen graph is (1,1,2,3)-colourable. In this paper we show using initially a similar approach as in [2] that every subcubic graph containing no Petersen graph as its component, is (1,1,2,3)-colourable. Since a (1,1,2,2)-colouring of a graph G does not neccesarly imply that G is also (1,1,2,3)-colourable (note that the converse is true), we remark that by proving every subcubic graph is (1,1,2,3)-colourable (with an exception of the Petersen graph), we have a stronger result of Theorem 3.2(If a graph G is a generalized prism of a cycle, then G is (1, 1, 2, 2)-colourable if and only if G is not the Petersen graph.) in [3].

[1] N. Gastineau and O. Togni. S-packing colorings of cubic graphs. *Discrete Mathematics*, 339:2461–2470, 2016.

[2] M. Jakovac and S. Klavžar. The b-chromatic number of cubic graphs. *Graphs and Combinatorics.*, 26:107–118, 2010.

[3] B. Brešar, S. Klavžar, D.F Rall and K. Wash. Packing chromatic number,[FORMULA]-colorings, and characterizing the Petersen graph. *Aequationes mathematicae.*, 91:169–184, 2017.

Mini-symposium
Emerging methods in Extremal Combinatorics Tuesday 1.50 Auditorium A

Forbidden vector-valued intersections

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University of Oxford

(This talk is based on joint work with Eoin Long.)

MSC2000: 05D05

We solve a generalised form of a conjecture of Kalai motivated by attempts to improve the bounds for Borsuk's problem. The conjecture can be roughly understood as asking for an analogue of the Frankl-Rödl forbidden intersection theorem in which set intersections are vector-valued. We discover that the vector world is richer in surprising ways: in particular, Kalai's conjecture is false, but we prove a corrected statement that is essentially best possible, and applies to a considerably more general setting. Our methods include the use of maximum entropy measures, VC-dimension, Dependent Random Choice and a new correlation inequality for product measures.

Counting Generalised Dyck Paths

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Queen Mary University of London

(This talk is based on joint work with Thomas Prellberg.)

MSC2000: 05Axx, 05Exx

In this work, we establish a general relationship between the enumeration of generalised Dyck paths and skew Schur functions, extending work by Bousquet-Mélou [1].

Generalised Dyck paths are directed lattice paths which can take steps out of a finite set of heights. We restrict these paths to be in a strip of height w and specify start-height u and end-height v. We further associate weights p_i to steps of height i, and denote the maximal height of an up-step and down-step by α and β, respectively.

Theorem 1. *The generating function $G^{w,\alpha,\beta}_{(u,v)}(t)$ of generalised Dyck paths is given by*

$$G^{w,\alpha,\beta}_{(u,v)}(t) = \frac{S_{(1^\alpha,0^\beta)}(\bar{z})}{S_{(w+1^\alpha,0^\beta)}(\bar{z})} S_{(w^\alpha,u,0^{\beta-1})/(v,0^{\alpha+\beta-1})}(\bar{z}) \ ,$$

where $S_{\lambda/\mu}(z)$ is a skew Schur function, and $\bar{z}$ are the $\alpha+\beta$ roots of the Kernel

$$K(t,z) = 1 - t\sum_{i=-\beta}^{\alpha} p_i z^i \ .$$

Note that this result separates the overall geometry of the problem from the detailed weights of the steps; the skew Schur functions are indexed by partitions depending on u, v, w, α and β, whereas the step weights only enter via the specification of the Kernel roots.

[1] M. Bousquet-Mélou, "Discrete Excursions.," *Séminaire Lotharingien de Combinatoire [electronic only]*, vol. 57, pp. B57d, 23 p., electronic only–B57d, 23 p., electronic only, 2006.

Shortest Path in Inner Dualist of Hexagonal Graph

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(This talk is based on joint work with Faqir M Bhatti[3].)

MSC2000: 05C85

A hexagonal graph is a graph created by joining different hexagonal blocks. This hexagonal graph is a planar graph. Its inner dual is defined by drawing a vertex for each hexagon. Vertices corresponding to adjacent hexagons in the original graph are connected through an edge in the inner dual. These edges make either an angle of π, $\pi/3$ or $2\pi/3$ radians in this particular setting.

The standard $(0, 1)$ matrix do not preserve the information about the angle of any edge in the inner dual. There are several methods to save this information and we use the He-Matrix. He-Matrix is an extension of adjacency matrix, where an entry of 0 means that the corresponding vertices are not connected directly by an edge. If there is an edge between two vertices then the entry in the matrix can be 1, 2 or 3 if the edge lie at an angle of π, $\pi/3$ or $2\pi/3$ radians respectively.

If the graph is rotated and reflected then there can be at most 6 non-isomorphic He-matrix of the graph. Each of them will correspond to a different orientation of the graph. In this talk we present some results about the shortest paths between two vertices based on He-Matrix. We discuss the combinatorial results on the number of shortest paths between any two given vertices in the inner dualist graph. We also present a linear time algorithm that solves the shortest path problem in this setting. Moreover, we find the orientation containing the smallest of all shortest path among all orientations.

[1]Khawaja M Fahd is a PhD Candidate

[3]Faqir M Bhatti is director of Riphah Institute of Computing and Applied Sciences, Riphah International University, Lahore

Characterizing excluded minor classes using the Strong Splitter Theorem

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MSC2000: 05B35

The Splitter Theorem states that, if N is a 3-connected proper minor of a 3-connected matroid M such that, if N is a wheel or whirl then M has no larger wheel or whirl, respectively, then there exists a sequence $M_0, \ldots, M_t$ of 3-connected matroids with $M_0 \cong N$, $M_t = M$, and for $i \in \{1, ...t\}$, M_i is a single-element extension or coextension of M_{i-1} [3]. The Strong Splitter Theorem optimizes the Splitter Theorem to best possible by showing that we can obtain up to isomorphism M starting with N and at each step performing a single-element extension or coextension, such that at most two consecutive single-element extensions occur in the sequence (unless the rank of the matroids involved is $r(M)$) [2]. The Strong Splitter Theorem is joint work with Manoel Lemos. In this talk I will describe how to use the Strong Splitter Theorem to characterize an excluded minor class of binary matroids [1].

[1] S. R. Kingan (submitted) On P_9^*-free matroids.

[2] S. R. Kingan and M. Lemos (2014) Strong Splitter Theorem, *Annals of Combinatorics*, Vol. 18-1, 111 - 116.

[3] P. D. Seymour (1980) Decomposition of regular matroids, *J. Combin. Theory Ser. B* **28**, 305 - 359.

Monday 1.45 Conference Room 6

Crown graphs and their representation numbers

Sergey Kitaev

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University of Strathclyde

(This talk is based on joint work with Marc Glen and Artem Pyatkin.)

MSC2000: 05C62

Letters x and y *alternate* in a word w if after deleting in w *all* letters *but* the copies of x and y we either obtain a word $xyxy\cdots$ (of even or odd length) or a word $yxyx\cdots$ (of even or odd length). A graph $G=(V,E)$ is word-representable if there exists a word w over the alphabet V such that letters x and y alternate in w if and only if xy is an edge in E. It is known [3] that any word-representable graph G is k-word-representable for some k, that is, there exists a word w representing G such that each letter occurs exactly k times in w. The minimum such k is called G's representation number.

A crown graph (also known as a cocktail party graph) $H_{n,n}$ is a graph obtained from the complete bipartite graph $K_{n,n}$ by removing a perfect matching. In this talk, I will discuss the results in [1], the main one of which is that for $n \geq 5$, $H_{n,n}$'s representation number is $\lceil n/2 \rceil$. This result not only solves the open Problem 7.4.2 in [2], but also gives a negative answer to the question raised in Problem 7.2.7 in [2] on 3-word-representability of bipartite graphs. As a byproduct, [1] gives a new example of a graph class with a high representation number.

[1] M. Glen, S. Kitaev, A. Pyatkin. On the representation number of a crown graph. arXiv:1609.00674

[2] S. Kitaev, V. Lozin. Words and graphs. Springer, 2015.

[3] S. Kitaev, A. Pyatkin. On representable graphs. *J. Autom., Lang. and Combin.* **13** (2008) 1, 45–54.

Variations on parking functions

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MSC2000: 05C30

A *parking function* is a sequence $S := (a_1, a_2, \ldots, a_n)$ of positive integers such that if $b_1 \le b_2 \le \ldots \le b_n$ is the increasing rearrangement of S, then $b_i \le i$ for all i. This corresponds to a queue of preferences of n cars entering a linear arrangement of n parking spots, where each car parks in its preferred spot if free, or the next available spot, otherwise (if any subsequent spots are available). Such a sequence is called a parking function if every car is able to park successfully. The definition was first introduced by Konheim and Weiss in 1966 [2]. H. Pollak gave an elegant "moonwalking"-proof of the fact that $(n+1)^{n-1}$ (compare to Cayley's formula) enumerates such functions [1]. Various authors have since given explicit bijections from the set of parking functions to the set of labelled $(n+1)$-vertex trees [3].

We will talk about the current state of the art regarding generalisations of parking functions (with more connections to graph theory), and we propose some variations.

[1] D. Foata and J. Riordan. Mappings of acyclic and parking functions. *Aequationes Mathématique*, 10:1022, 1974.

[2] A. G. Konheim and B. Weiss. An occupancy discipline and applications. *SIAM Journal on Applied Mathematics*, 14(6):12661274, 1966.

[3] G. Kreweras. Une famille de polynômes ayant plusieurs propriétés énumeratives. *Period. Math. Hungar.*, 11(4):309320, 1980.

Uniqueness of optimal configurations in extremal combinatorics

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(This talk is based on joint work with Andrzej Grzesik and László Miklós Lovász.)

MSC2000: 05C35

We study the uniqueness of optimal configurations in extremal combinatorics. An empirical experience suggests that optimal solutions to extremal graph theory problems can be made asymptotically unique by introducing additional constraints. Using tools from the theory of graph limits, we will show that this phenomenon is not true in general. In particular, we will present a counterexample to the following conjecture of Lovász, which is often referred to as saying that "every extremal graph theory problem has a finitely forcible optimum": every finite feasible set of subgraph density constraints can be extended further by a finite set of density constraints such that the resulting set is satisfied by an asymptotically unique graph.

Violator spaces vs closure spaces

Vadim E. Levit

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Ariel University, Israel

(This talk is based on joint work with Yulia Kempner.)

MSC2000: 05B35

The main goal of this talk is to make connections between two well-known, but, up to now, independently developed theories: the theory of violator spaces and the theory of closure spaces.

LP-type problems have been introduced and analyzed by Matoušek, Sharir and Welzl as a combinatorial framework that encompasses linear programming and other geometric optimization problems. Further, Matoušek et al. [1] defined a more general framework: violator spaces, which constituted a proper generalization of LP-type problems. Originally, violator spaces were defined for the set of constraints H, where with each subset of constraints $G \subseteq H$ associates $V(G)$ - the set of all constraints violating G. For instance, a violator space is naturally revealed, when one computes the smallest enclosing ball of a finite set of points in R^d. Here the set H is a set of points in R^d, and the violated constraints of some subset of the points G are exactly the points lying outside the smallest enclosing ball of G.

Convex geometries were invented by Edelman and Jamison [2] as proper combinatorial abstractions of convexity. There are various ways to characterize finite convex geometries. One of them defines convex sets by an anti-exchange closure operator. The convex hull operator on Euclidean space E^n is a classic example of a closure operator with the anti-exchange property.

In this work, we investigate interrelations between violator spaces and closure spaces and show that a violator mapping may be defined by a weakened version of a closure operator. Interrelations between violator spaces and closure spaces gives new insights on a number of well known findings. For example, we prove that violator spaces with a unique basis satisfy both the anti-exchange and the Krein-Milman properties.

Finally, based on subsequent relaxations of the closure operator notion we introduce a notion of a *convex space* as a generalization of a violator space.

[1] Gärtner, J. Matoušek, L. Rüst, P. Škovroň. Violator spaces: structure and algorithms. *Discrete Appl. Math.*, 156 (11): 2124 – 2141, 2008.

[2] P.H. Edelman and R.E. Jamison. The theory of convex geometries, *Geom. Dedicata*, 19, 247-270, 1985.

Inverse of the Pak-Stanley bijection for k-Shi arrangement

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(This talk is based on joint work with Stefan Heuer.)

MSC2000: 05A15

For positive integers k, n, the *n-dimensional k-Shi arrangement* consists of all hyperplanes of the form

$$x_i - x_j = -k+1, -k+2, \dots, -1, +, 1, \dots, k-1, k \quad \text{for all } 1 \le i < j \le n.$$

When $k = 1$ it is called the Shi arrangement. The number of regions of the k-Shi arrangement is $(kn+1)^{n-1}$.

A *k-parking function* of length n is a sequence of positive integers $\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{Z}^n_{>0}$ that is component-wise less than or equal to some permutation of $(b_1, \dots, b_n)$ where $b_i = 1 + k(i-1)$ (see [3]). The 1-parking functions are the ordinary parking functions. For example $(9, 7, 1, 2)$ is a 3-parking function of length 4, but not a 2-parking function.

Pak and Stanley defined a natural bijection from the regions of the k-Shi arrangement to k-parking functions of length n, see [3]. The map carried important statistics and was clearly injective, but had the drawback that they could not formulate an explicit inverse. Their proof of bijectivity used the fact that the sets were known to be equinumerous. In a recent paper [1] Beck et al. defines an explicit inverse for the case of $k = 1$.

We have defined an explicit inverse to the Pak and Stanley map for general k, that is, a map from k-parking functions to regions of the k-Shi arrangement. Our map goes via objects called k-parking graphs, a generalisation of the parking graphs considered by Beck et al. Detailed proofs can be found in the thesis of Stefan Heuer [2].

[1] Matthias Beck, Ana Berrizbeitia, Michael Dairyko, Claudia Rodriguez, Amanda Ruiz and Schuyler Veeneman, "Parking functions, Shi Arrangements, and Mixed Graph", *The American Mathematical Monthly* vol 122, no 7 (2015), pp. 660?673.

[2] Stefan Heuer, "Bijections between k-Shi arrangement, k-parking functions and k- parking graphs", http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-173800, master thesis. KTH, Sweden.

[3] Rickard P. Stanley, "Hyperplane arrangements, parking functions and tree inversions", In: Mathematical Essays in Honor of Gian-Carlo Rota. Ed. by B. E. Sagan and R. P. Stanley. Vol. 161. Progress in Mathematics. Birkhäuser Boston, 1998, pp. 2620 – 2625.

Designs beyond quasirandomness

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(This talk is based on joint work with Stefan Glock, Daniela Kühn and Deryk Osthus.)

MSC2000: 05C70, 05C65, 05B40

The Existence conjecture for combinatorial designs states that every large complete k-graph can be edge-decomposed into small cliques (subject to the obvious divisibility condition). This conjecture was proved by a recent breakthrough of Keevash. We gave a new proof of this result, based on the method of iterative absorption. In fact, 'regularity boosting' allows us to extend our main decomposition result beyond the quasirandom setting and thus to generalise the results of Keevash. In particular, we obtain a resilience version and a minimum degree version. In this talk, we will present our new results within a brief outline of the history of the Existence conjecture and provide an overview of the proof.

Tournament Quasirandomness from counting

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(This talk is based on joint work with Asaf Shapira and Benny Sudakov.)

MSC2000: 05C35

A theorem of Chung and Graham states that if $h \geq 4$ then a tournament T is quasirandom if and only if T contains each h-vertex tournament the 'correct number' of times as a subtournament. Here we investigate the relationship between quasirandomness of T and the count of a *single* h-vertex tournament H in T. We show that if T has the correct global count of H and $h \geq 7$ then quasirandomness of T is only forced if H is transitive. However if T furthermore has the correct local count of H in all large subsets of the vertex set of T then one can sometimes say more. We prove that although this stronger property does not imply quasirandomness of T for many tournaments H, it does suffice for an infinite collection.

Characterizing path-like trees from linear configurations

Susana-Clara López

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(This talk is based on joint work with Francesc-Antoni Muntaner-Batle.)

MSC2000: Primary 05C05, 05C75, Secondary 05C70 and 05C78

Assume that we embed the path P_n as a subgraph of a 2-dimensional grid, namely, $P_k \times P_l$. Given such an embedding, we consider the ordered set of subpaths $L_1, L_2, \ldots, L_m$ which are maximal straight segments in the embedding, and such that the end of L_i is the beginning of L_{i+1}. Suppose that $L_i \cong P_2$, for some i and that some vertex u of L_{i-1} is at distance 1 in the grid to a vertex v of L_{i+1}. An elementary transformation of the path consists in replacing the edge of L_i by a new edge uv. A tree T of order n is said to be a path-like tree, when it can be obtained from some embedding of P_n in the 2-dimensional grid, by a sequence of elementary transformations. Thus, the maximum degree of a path-like tree is at most 4.

Intuitively speaking, a tree admits a linear configuration if it can be described by a sequence of paths in such a way that only vertices from two consecutive paths, which are at the same distance of the end vertices are adjacent. In this work, we characterize path-like trees of maximum degree 3, with an even number of vertices of degree 3, from linear configurations. We also show that the characterization of path-like trees of maximum degree 4 can be reduced to the characterization of path-like tree of maximum degree 3.

[1] S.C. López, F. A. Muntaner-Batle, Characterizing path-like trees from linear configurations, arXiv:1705.08802 [math.CO].

Snarks that cannot be covered with four perfect matchings

Edita Máčajová

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Comenius University, Bratislava, Slovakia

(This talk is based on joint work with Martin Škoviera.)

MSC2000: 05C70, 05C15

The celebrated Berge-Fulkerson conjecture suggests that every bridgeless cubic graph can have its edges covered with at most five perfect matchings. Since three perfect matchings suffice if and only if the graph in question is 3-edge-colourable, uncolourable cubic graphs fall into two classes: those that can be covered with four perfect matchings, and those that require at least five. Cubic graphs that cannot be covered with four perfect matchings are extremely rare. Among the 64326024 snarks (uncolourable cyclically 4-edge-connected cubic graphs with girth at least five) on up to 36 vertices there are only *two* graphs that cannot be covered with four perfect matchings – the Petersen graph and a snark of order 34.

In this talk we show that coverings with four perfect matchings can be described with the help of nowhere-zero flows whose values lie in the configuration of six lines spanned by four points of the 3-dimensional projective geometry $PG(3,2)$ in general position. This characterisation provides a convenient tool for investigation of graphs that do not admit such a cover and enables a great variety of constructions of snarks that cannot be covered with four perfect matchings. In particular, with the combined forces of several constructions we can prove that for each even integer $n \geq 44$ there exists at least one snark of order n that has no cover with four perfect matchings.

Thursday 4.45 Auditorium A

The Erdős-Ko-Rado Theorem, Generalisations and Beyond

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MSC2000: 05D05

Extremal Set Theory is a branch of Extremal Combinatorics where one characterises the maximum size of families of sets with given restrictions placed on them. The Erdős-Ko-Rado Theorem is a classical result in Extremal Set Theory that has, since its discovery, been extensively researched and generalised.

In this talk, I will present an introduction of the Erdős-Ko-Rado Theorem and some of its generalisations. I also will present some of my own results which relate to a particular generalisation of this theorem. Open problems as well as new possible directions of research are also given.

Quantum and non-signalling graph isomorphism

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(This talk is based on joint work with Albert Atserias, David Roberson, Robert Šámal, Simone Severini, Antonios Varvitsiotis.)

MSC2000: 05C60

We introduce a two-player nonlocal game, called the (G, H)-*isomorphism game*, where classical players can win with certainty if and only if the graphs G and H are isomorphic. We then define the notions of quantum and non-signalling isomorphism, by considering perfect quantum and non-signalling strategies for the (G, H)-isomorphism game, respectively. In the quantum case, we consider both the tensor product and commuting frameworks for nonlocal games. We prove that non-signalling isomorphism coincides with the well-studied notion of fractional isomorphism, thus giving the latter an operational interpretation. Second, we show that, in the tensor product framework, quantum isomorphism is equivalent to the feasibility of two polynomial systems in non-commuting variables, obtained by relaxing the standard integer programming formulations for graph isomorphism to Hermitian variables. On the basis of this correspondence, we show that quantum isomorphic graphs are necessarily cospectral. Finally, we provide a construction for reducing linear binary constraint system games to isomorphism games. This allows us to produce quantum isomorphic graphs that are nevertheless not isomorphic. Furthermore, it allows us to show that our two notions of quantum isomorphism, from the tensor product and commuting frameworks, are in fact distinct relations, and that the latter is undecidable. Our construction is related to the FGLSS reduction from inapproximability literature, as well as the CFI construction. The techniques used are an interesting mix of combinatorics, optimization, and quantum information.

This talk will be based on arXiv:1611.09837.

Intersection of transversals in the Latin square B_n, with applications to Latin trades

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(This talk is based on joint work with Ji Lijun.)

MSC2000: 05B15

A paper by Cavenagh and Wanless diagnosed the possible intersection of any two transversals of B_n. We give a generalization of this problem for the intersection of μ transversals, and provide constructions and computational results for the cases where $\mu = 3, 4$. This result is then applied to the problem of finding μ-way k-homogeneous Latin trades, and along with a few new constructions, completes the spectrum of the existence 3-way k-homogeneous Latin trades for all but a small list of exceptions.

Condorcet domains and Turán type problems for permutations

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(This talk is based on joint work with Søren Riis and Charles Leedham-Green.)

MSC2000: 05A05, 06A05, 91B12

A Condorcet domain is a set of linear orders which give rise to a transitive set of preferences when used as rankings in majority voting. Alternatively they can be seen a a set of permutations such that any three permutations do not induce a certain forbidden pattern. In this talk I'll give some background on Condorcet domains and describe some results from a project together with Søren Riis and Charles Leedham-Green. I will also relate these results to a more general type of Turán type problem for permutations, which also includes the classical theory of pattern avoiding permutations.

List edge-colouring graphs with restricted odd cycles

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(This talk is based on joint work with Gregory J. Puleo.)

MSC2000: 05C15

In this talk we'll look at the class of simple graphs $\mathcal{G}^*$ for which every pair of distinct odd cycles intersect in at most one edge. We'll give a structural characterization of these graphs, and prove that they satisfy the list-edge-colouring conjecture. We will also talk about a stronger result concerning kernel-perfect orientations in line graphs.

The parameterised complexity of computing the maximum modularity of a graph

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(This talk is based on joint work with Fiona Skerman, University of Bristol.)

MSC2000: 05C85

A central topic in the study of complex networks is the comparison of the amount of clustering or community structure between different networks, and one of the most popular ways to quantify the amount of clustering in a network is through its *modularity*. Given a graph $G = (V, E)$ where $|V| = n$ and $|E| = n$, the *maximum modularity* of G is defined to be the maximum, taken over all partitions $\mathcal{A}$ of V, of

$$q_{\mathcal{A}}(G) = \frac{1}{m} \sum_{A \in \mathcal{A}} e(A) - \frac{1}{4m^2} \sum_{A \in \mathcal{A}} \left(\sum_{v \in A} d(v) \right)^2 .$$

While a range of heuristics are used in practice to estimate this quantity, it is notoriously hard to compute exactly: in fact it is even NP-hard to approximate the maximum modularity within any constant factor.

In this talk I will present some initial results concerning the parameterised complexity of this problem, with respect to some standard structural parameters. On the positive side, we show that the problem belongs to FPT when parameterised by the size of a minimum vertex cover for the input graph. However, although the problem can be solved in polynomial time on graphs whose treewidth is bounded by a fixed constant, we provide evidence that the problem is unlikely to be in FPT with respect to this parameterisation.

Perfect state transfer in Cayley graphs over finite chain rings

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(This talk is based on joint work with Issaraporn Thongsomnuk.)

MSC2000: 05A05, 05C25

Recently, Suntornpoch and Meemark [8] studied the Cayley graphs over finite chain rings as a generalisation of integral circulant graphs and GCD-graphs with prime power order presented in [4, 5, 7]. They computed its eigenvalues and showed that this Cayley graph is an integral circulant graph. In this talk, we consider the problem on existence of perfect state transfer in these graphs. For some integral circulant graphs, the problem of the existence of perfect state transfer is known, e.g., [1, 2, 3, 6]. Bašić et al. [1] showed that integral circulant graphs having no perfect state transfer if the set of divisor contain only odd divisors. They also gave the simple condition for characterizing integral circulant graphs allowing perfect state transfer in terms of its eigenvalues. As we know that the Cayley graphs over finite chain rings are indeed integral circulant and we know their eigenvalues, we can apply Bašić's tool on the eigenvalues to determine the existence of perfect state transfer in these graphs.

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Extremal multicomplexes

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(This talk is based on joint work with Pedro Antonio Soto.)

MSC2000: 05E45

A non-empty set of monomials M is a multicomplex if, for a monomial m in M and a monomial m' such that $m'|m$, we have that m' also belongs to M. A multicomplex M is called pure if all its maximal elements are of the same degree. This notion is clearly a generalization of the simplicial complex, and several invariants extend directly, as the f-vector of a multicomplex, which is the vector that lists the monomials grouped by degrees.

The combinatorial relevance of multicomplexes is partly due to a 1977 Richard Stanley conjecture that says that the h-vector of a matroide is the f-vector of a pure multicomplex. This has been proved for several families of matroids. In [1], the conjecture is proved for paving matroids.

Let R be the ring $k[X_0, ..., X_{n-1}]$, where k is a field, and let us consider the set $G_{d,n}$ of monomials in R of degree d and coefficient 1. We can give to $G_{d,n}$ the structure of a graph as follows : Two monomials m and m' are adjacent if there are different integers i and j such that $X_i m = X_j m'$.

In this talk, we will consider two conjectures about certain invariants of these graphs related to pure pure multicomplexes and some algebraic properties of R. These conjectures appear in [1] and the invariants in [2].

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Statistics on Set Partitions

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MSC2000: 05A15

Given a positive integer n, we denote a set partition of $I_n := \{1, 2, \ldots, n\}$ into k nonempty parts by $P = B_1/B_2/\cdots/B_k \in \Pi_{n,k}$, where $min(B_1) < min(B_2) < \cdots < min(B_k)$. For example, $P = 147/238/56 \in \Pi_{8,3}$. It is well known that $\left\{ {n \atop k} \right\} = |\Pi_{n,k}|$ is the *Stirling number of the second kind*, which for $0 \leq k \leq n$ satisfies the recursion

$$\left\{ {n+1 \atop k} \right\} = \left\{ {n \atop k-1} \right\} + k \left\{ {n \atop k} \right\},$$

wth initial conditions $\left\{ {0 \atop 0} \right\} = 1$ and $\left\{ {n \atop k} \right\} = 0$ if $n, k < 0$ or $n < k$. Over the past few decades, various generalizations of $\left\{ {n \atop k} \right\}$ have appeared in the literature, and the aim of this talk is to give alternative ways of partitioning I_n, along with their combinatorial connections to other objects.

For instance, we say j is a *fixed point* of a partition $P = B_1/B_2/\cdots/B_k$ of I_n if $j \in B_j$, and we define $S_{fix}(n, r)$ to be the number of partitions of I_n with exactly r fixed points. Then $S_{fix}(n, r)$ corresponds in a natural way to counting Broder's [1] partitions of I_n into k parts such that $1, 2, \ldots, r$ are in distinct parts, known as *r-Stirling numbers of the second kind*, $\left\{ {n \atop k} \right\}_r$. Another generalization is $S_{min}(n, i)$, defined to be the number of partitions of I_n such that i is a minimal element of some block B_ℓ. The $S_{min}(n, i)$ correspond to many combinatorial objects, including the $1\underline{32}$-avoiding indecomposable permutations studied by Gao, Kitaev, and Zhang [2].

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The Tutte polynomial and its extensions

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MSC2010: 05C35

Arguably, the most important and best studied graph polynomial is the Tutte polynomial. It is important not only because it encodes a large amount of combinatorial information about a graph, but also because of its applications to areas such as statistical physics and knot theory. Because of its importance the Tutte polynomial has been extended to various other classes of combinatorial object. For some objects there is more than one definition of a "Tutte polynomial". For example, there are three different definitions for the Tutte polynomial of graphs in surfaces: M. Las Vergnas' 1978 polynomial, B. Bollobás and O. Riordan's 2002 ribbon graph polynomial, and V. Kruskal's polynomial from 2011. On the other hand, for some objects, such as digraphs, there is no wholly satisfactory definition of a Tutte polynomial. Why is this? Why are there three different Tutte polynomials of graphs in surfaces? Which can claim to be *the* Tutte polynomial of a graph in a surface? More generally, what does it mean to be the Tutte polynomial of a class of combinatorial objects? In this talk I will describe a way to canonically construct Tutte polynomials of combinatorial objects, and, using this framework, will offer answers to these questions.

Plenary lecture

Tuesday 11.15 Main Auditorium

Monotone cellular automata

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(This talk is based on joint work with Paul Balister, Béla Bollobás, Hugo Duminil-Copin, Fabio Martinelli, Paul Smith, and Cristina Toninelli.)

MSC2000: 60C05

Cellular automata are interacting particle systems whose update rules are homogeneous and local. Since their introduction by von Neumann almost 50 years ago, many particular such systems have been investigated, but no general theory has been developed for their study, and for many simple examples surprisingly little is known. Understanding the rules that govern their typical global behaviour is an important and challenging problem in statistical physics, probability theory and combinatorics.

In this talk we will discuss some dramatic recent developments in our understanding of a particular (large) family of *monotone* cellular automata – those which can naturally be embedded in d-dimensional space – with random initial conditions. This family was first studied by Bollobás, Smith and Uzzell, and is a substantial generalization of bootstrap percolation, which corresponds to the case in which a site updates (from inactive to active) if at least r of its neighbours are already active. We will also discuss some very recent applications to so-called 'kinetically constrained spin models'.

Hypergraphical Matroids

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MSC2000: 05A15, 05B35, 05C65

In graph theory some of the most important objects are spanning trees and forests. For usual graphs there are a lot of famous enumeration results of trees. In this work we focus on Tutte polynomials and graphical matroids, which "enumerate" spanning trees and forests. We will discuss the generalization of spanning trees and forests for the case of hypergraphs. Then we will introduce the concept of hypergraphical matroid for an arbitrary hypergraph. Our definition of trees looks similar to the definition given in [1]. However our definition allows to associate a matroid to a hypergraph, when the definition given in [1] allows to associate a polymatroid (they are not necessarily matroids).

For a hypergraph $H = (V, E)$, a subset of edges $C \subset E$ is called a *cycle* if $|C| = |\cup_{e \in C} e|$ and there is no subset $C' \subsetneq C$, such that the first property holds for C'. Of course, it gives the definition of forests and also of trees (forests of size $|V| - 1$). Define the *hypergraphical matroid* of H as the matroid with the ground set E and with cycles as minimal dependents.

This definition gives the matroid, so we have all matroid properties of forests, for example: maximal forests have the same cardinality; the Tutte polynomial, which enumerates trees and forests. Furthermore, I will present another definition of a hypergraphical matroid, which is equivalent to the first and which obviously gives a vector matroid. The second definition is motivated by Postnikov-Shapiro algebras (power algebras associated to graphs, see [3]), which is in one to one correspondence with graphical matroids, see [2].

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A NEW FAMILY OF TRIPLE ARRAYS

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(This talk is based on joint work with Peter J. Cameron.)

MSC2000: 05B05, 05B10, 05B15, 05B30

An $r \times c$ *triple array* on v symbols is an array in which each symbol occurs equally often, with no repeats in rows or columns, and the number of symbols common to two rows, two columns, or a row and a column are (possibly different) constants.

Agrawal [1] suggested a method for constructing triple arrays from symmetric 2-designs, giving rise to what is called *Agrawal's conjecture*

Conjecture 1. *If there is a symmetric* 2-$(v+1, r, \lambda)$ *design with* $r - \lambda > 2$*, then there is an* $r \times c$ *triple array on* v *symbols with* $v = r + c - 1$*.*

The converse of this conjecture was proven in [2] and many examples have been constructed. But, until now only one infinite family, called *Paley triple arrays*, has been proven to exist. This has been done to different degrees by various authors over the years, but to the full extent in [4], and can be summarized as follows.

Theorem 2. *Let* $q \geq 5$ *be an odd prime power. Then there exists a* $q \times (q+1)$ *triple array.*

In this talk, we introduce a new infinite family of triple arrays. The construction is based on the use of difference sets in abelian groups which admit -1 (the inverse mapping $x \mapsto x^{-1}$) as a so-called *multiplier*, and this property turn out to be both necessary and sufficient.

For difference sets in non-abelian groups, -1 can only be a *weak multiplier*, and this is not sufficient. We have not found any examples of difference sets in non-abelian groups that give triple arrays by this construction.

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Alternating Signed Bipartite Graphs

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(This talk is based on joint work with Rachel Quinlan and Kevin Jennings.)

MSC2000: 05C50

Alternating Sign Matrices (ASMs) are fascinating combinatorial objects with connections to many different areas of science. Associated to any ASM is its Alternating Signed Bipartite Graph, and these have been the main focus of our research. One particular question of interest asks which balanced 2-edge-coloured bipartite graphs admit realisations as alternating sign matrices.

An Alternating Sign Matrix, or ASM, is an $n \times n$ matrix that contains only the numbers 0, 1, and -1, subject to the following constraints:

- The sum of each row and column must be 1
- The non-zero entries in each row must alternate between 1 and -1
- The non-zero entries in each column must alternate between 1 and -1

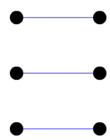
Associated to each ASM is its Alternating Signed Bipartite Graph. This graph has a vertex for each row and column of the matrix, and the vertices belonging to Row i and Column j are adjacent if the (i, j) entry of the matrix is non-zero. Edges are coloured with two colours according as they represent positive (blue) or negative (red) matrix entries.

Here are all of the 3×3 ASMs, and their corresponding ASBGs:

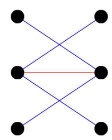
$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$



$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$



Some basic criteria that the graph must meet in order to be isomorphic to an ASBG are:

- The graph must be bipartite.
- This bipartition must be "balanced". In other words, there must be the same number of vertices in each part of the bipartition.

These criteria alone, however, are not enough to guarantee that a given graph is an ASBG. What we would like is a set of criteria that a given graph can either pass or fail, in order to determine if they are isomorphic to an ASBG. This talk will present partial characteristics of ASBGs.

[1] R. Brualdi, K. Kiernan, S. Meyer, M. Schroeder. Patterns of Alternating Sign Matrices. *Linear Algebra and its Applications*,2013.

Edge-minimal graphs of exponent 2

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(This talk is based on joint work with Rachel Quinlan.)

MSC2000: 05C35

A simple undirected graph G has the Friendship property if every pair of distinct vertices in G have exactly one mutual neighbour. The Erdős-Rényi-Sós theorem, commonly referred to as the Friendship Theorem, establishes that the only finite graphs with the Friendship property are the windmill graphs.

We will discuss a relaxation of the friendship property, namely the *minimal exponent* 2 *(me$_2$)* property. An me$_2$-graph is a simple undirected graph of exponent 2 in which the deletion of any edge would result in a graph not having exponent 2. If u and v are adjacent vertices in an me$_2$-graph G, then either u is the unique common neighbour in G of v and another vertex w, or v is the unique common neighbour in G of u and another vertex w'. If both of these properties hold for every pair of adjacent vertices in G, then we say that G has the *strong-me$_2$* property. This talk will focus on the me$_2$ and strong-me$_2$ properties and the relationship between them. In a strong-me$_2$ graph of order n, the maximum possible vertex degree is $n-1$ if n is odd, and $n-4$ if n is even. The examples of odd order in which this maximum is attained are the windmills; a description of the examples of even order will be presented.

DOMINATING SET RECONFIGURATION

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(This talk is based on joint work with Marthe Bonamy and Paul Dorbec.)

MSC2000: 05C69

Reconfiguration problems arise when, given an instance of a combinatorics problem, we want to transform step-by-step a solution into another one without losing the desired property at any time. Each step is defined by an elementary operation. Three elementary transformations, called reconfiguration rules, are usually considered. They are naturally explained using tokens placed on the vertices that form the solution. In the *token addition and removal* model (TAR), we can add or remove a token as long as the number of tokens does not go beyond a given threshold. In the *token jumping* model (TJ), we can move a token to any vertex of the graph. Finally, in the *token sliding* model (TS), we can slide a token along an edge (i.e. we move the token to a neighbour of its current location). Note that in the TJ and TS models, the size of a solution does not change under an elementary operation.

Here, we are interested in reconfiguring dominating sets under token sliding. Given a graph $G = (V, E)$, a dominating set of G is a subset of vertices $D \subseteq V$ such that the closed neighbourhood of D (denoted $N_G[D]$) satisfies $N_G[D] = V$. We define the DOMINATING SET RECONFIGURATION (DSR) problem as that of, given two dominating sets, deciding whether they can be reconfigured into each other. To this day, many reconfiguration results consider COLOURING or INDEPENDENT SET (see [2] for a survey) and there are still few results in the context of dominating set reconfiguration. Haddadan et al. [1] studied this problem under the TAR model. They proved that it is PSPACE-complete even for planar graphs, bounded treewidth graphs or bipartite graphs. On the other hand, they gave polynomial-time algorithms for interval graphs, trees and cographs.

We investigate the complexity of DSR under TS. We prove that this problem is PSPACE-complete even for planar graphs, bounded treewidth graphs or split graphs, and provide polynomial-time algorithms for interval graphs and dually chordal graphs.

[1] Arash Haddadan, Takehiro Ito, Amer E. Mouawad, Naomi Nishimura, Hirotaka Ono, Akira Suzuki, and Youcef Tebbal. *The Complexity of Dominating Set Reconfiguration*, pages 398–409. Springer International Publishing, Cham, 2015.

[2] Jan van den Heuvel. The complexity of change. *CoRR*, abs/1312.2816, 2013.

On the existence and construction of PD–sets for linear codes

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MSC2000: 94B05

Linear error-correcting codes with large automorphism groups are of interest from many points of view. From the Coding Theory point of view, a large automorphism group can reduce the number of computations needed for encoding and decoding. In [1], MacWilliams developed a method, called permutation decoding, that depends on the existence of a particular set of automorphisms, called PD–set. A PD–set for a t–error correcting code $\mathcal{C}$ is a set $\mathcal{S}$ consisting of automorphisms of the code such that every possible error vector of weight at most t can be moved outside the information part by some element of $\mathcal{S}$.

Let $G = Aut(\mathcal{C})$ be the automorphism group of $\mathcal{C}$. There are two practical problems:

(1) determine whether G contains a PD–set or not;

(2) if G contains a PD–set, then construct a PD-set of smallest possible size.

These problems were studied for very particular classes of codes and not much is known for the general case. In particular, no efficient algorithm is known for either (1) or (2). In this talk, we will consider several linear codes constructed in [2, 3] and illustrate some possible computational approach to problems (1) and (2).

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Nonexistence results for strong external difference families

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(This talk is based on joint work with Sophie Huczynska.)

MSC2000: 94C30

An $(n, m, k; \lambda)$-strong external difference family (SEDF) is a set of m disjoint k-subsets $A_1, \ldots, A_m$ of an additive group G of order n with the property that for each i from 1 to m we have that we have that each nonzero element of G occurs precisely λ times as a difference of the form $a_i - a_j$ with $a_i \in A_i$ and $a_j \in A_j$ for some $j \neq i$. Their study is motivated by a connection with algebraic manipulation detection codes, which have applications in cryptography and coding theory. In this talk we discuss counting techniques that give nonexistence results for SEDFs for many choices of the parameters n, m, k and λ.

Negatively invertible graphs

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(Based on a joint work with Daniel Ševčovič)

MSC2000: 05C50

Following Godsil [1], a graph is (positively) *invertible* if it has a non-singular adjacency matrix the inverse of which is diagonally similar to a non-negative integral matrix. This inverse is then the adjacency matrix of a (multi)graph, called the *inverse* of the original graph. Interest in inverse graphs is motivated by the fact that their spectra are reciprocal to those of the original graphs; for other properties and applications we refer to [1,2,3].

In our talk we will focus on a negative counterpart of invertibility as introduced in [4]. Namely, we call a graph *negatively invertible* if it has a non-singular adjacency matrix the inverse of which is diagonally similar to a non-positive integral matrix M. The *negative inverse graph* is then the one with the adjacency matrix $-M$; its spectrum is negatively reciprocal to the spectrum of the original graph.

Although the two concepts are defined similarly, they lead to distinct sets of invertible graphs. It also turns out that the family of negatively invertible graphs contains representatives of structural models of important molecules and such graphs also appear in other applications.

As a taster we present a census of positively and negatively invertible graphs on at most 6 vertices with a unique perfect matching. Among out main results we propose a construction of invertible graphs in either sense, based on 'bridging' two invertible graphs over a subset of their vertices. We also analyze spectral properties of bridged graphs and derive lower bounds for their least positive eigenvalues.

References:

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Tropical paths in vertex-colored graphs

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(This talk is based on joint work with Yannis Manoussakis, Giuseppe F. Italiano, Thang Nguyen Kim, Johance Cohen, Katerina Bhmova.)

MSC2000: 05C38

Given a vertex-colored graph, a tropical subgraph is defined as a subgraph where each color of the initial graph appears at least once. Applications of vertex-colored graphs can be found a lot in bioinformatics and web graphs. In practice, some ongoing works for tropical subgraphs problems are tropical dominating sets, tropical connected graphs, tropical maximum matchings and most of them are NP-hard. In this work, we deal with the problem of looking for *tropical paths* in a vertex-colored graph. It is well known that the longest path problem is harder than the Hamiltonian path problem and a very limited number of classes of graphs can be efficiently solved for the former. Note that our problem of tropical paths is even harder than the longest path problem as we color each vertex by a distinct color. Thus we try to focus on the classes of graphs which have been deeply studied for the longest path problem, to apply for the tropical path problem. We show that some graph classes which are easy for the longest path problem remains NP-hard for our problem such as DAG, interval, cactus graphs, etc, especially our problem is still NP-hard if allowing to visit each vertex more than once on paths in general graphs. Besides, we find out classes of graphs which are solved in polynomial time for our problem and we also propose some efficient algorithms. Finally, we also arise some open questions for this hard problem.

Twofold triple systems with 2-intersecting Gray codes

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(This talk is based on joint work with Aras Erzurumluoğlu.)

MSC2000: 05B05, 05B07, 05C38

A λ-fold triple system of order v is a design consisting of a v-set V and a collection of 3-subsets (called blocks or triples) of V such that each 2-subset of V occurs in exactly λ of the triples of the design. Given a λ-fold triple system with $\lambda > 1$, we can ask whether its triples can be ordered so that the union of any two consecutive triples consists of four elements of V; when this is possible we have a 2-intersecting Gray code for the design. We will describe some potential applications, give a review of previous existence and non-existence results, and discuss some recent advances concerning the existence of 2-intersecting Gray codes for twofold triple systems.

On the sum of Laplacian spectra of graphs

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(This talk is based on joint work with Hilal A. Ganie.)

MSC2000: 05C30, 05C50

Let G be a simple graph with n-vertices, m edges and having Laplacian eigenvalues $\mu_1, \mu_2, \ldots, \mu_{n-1}, \mu_n = 0$. Let the sum of the k largest Laplacian eigenvalues of G be $S_k(G) = \sum_{i=1}^{k} \mu_i$. Brouwer conjectured that $S_k(G) \leq m + \binom{k+1}{2}$, for all $k = 1, 2, \ldots, n$. We obtain upper bounds for $S_k(G)$ in terms of the clique number ω, the vertex covering number τ and the diameter d of a graph G. We show that Brouwer's conjecture holds for certain classes of graphs. As an application, we obtain some upper bounds for Laplacian energy $LE(G)$ which are more stronger than the existing bounds.

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Construction of Pairs of Orthogonal Latin Cubes

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MSC2000: 05B05, 05B15

A *latin square* of order n is an $n \times n$ array of n symbols in which each symbol occurs exactly once in every row and in every column. A d-dimensional array satisfying the same condition is called a *latin d-cube*. Two latin squares are *orthogonal* if, when they are superimposed, every ordered pair of symbols appears exactly once. If in a set of latin squares, any two Latin squares are orthogonal then this set is called a system of Mutually Orthogonal Latin Squares (MOLS). From the definition we can ensure that a latin d-cube is the Cayley table of a d-ary quasigroup. A system consisting of t s-ary functions $f_1, \dots, f_t$ with domain Q^s $(t \geq s)$ is *orthogonal*, if for each subsystem $f_{i_1}, \dots, f_{i_s}$ consisting of s functions it holds $\{(f_{i_1}(\overline{x}), \dots, f_{i_s}(\overline{x})) \mid \overline{x} \in Q^s\} = Q^s$. If the system keeps to be orthogonal after substituting any constants for each subset of variables then it is called *strongly orthogonal*. If the number of variables is two, then such system is a system of MOLS (see [1]). If $s = 3$, it is a set of Mutually Orthogonal Latin Cubes (MOLC).

Let $Q = F_q$ be a Galois field of order q. A system of t MOLC is equivalent to an MDS code with distance $t+1$ (see [1]). Hence, there exists a linear (over F_q) system consisting of t Mutually Orthogonal Latin s-Cubes as $t + s \leq q + 1$ (see [2]). Cartesian product of two strongly orthogonal systems over alphabets Q_1 and Q_2 is a strongly orthogonal system over the alphabet $Q_1 \times Q_2$. Thus we obtain that if $\delta_2 \neq 1$ and $\delta_3 \neq 1$ where $q = 2^{\delta_2} 3^{\delta_3} 5^{\delta_5} \dots$ is the factorization, then there exists a pair of orthogonal latin cubes of order q. It is well known that pairs of MOLS of order q do not exist if and only if $q = 2, 6$. For all without a finite number of orders we can use Wilson's construction to obtain pairs of MOLS. But for pairs of MOLC such constructions were unknown. Now we present a Wilson-like construction for pairs of MOLC. By using new construction we obtain

Proposition 1. *If $q = 16(6k \pm 1) + 4$ then there exists a pair of MOLC of order q.*

If $6k - 1 = 18i - 1$ or $6k - 1 = 18i + 5$ then pairs of MOLC of order $q = 16(6k - 1) + 4$ was not previously known. A minimal new order is 84.

The work was funded by the Russian Science Foundation (grant No 14-11-00555).

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Counting matrices over finite fields

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(This talk is based on joint work with Cian O'Brien and Ha Van Hieu.)

MSC2000: 15B33

The *stable rank* of a $n \times n$ matrix A is the rank of A^n. The stable rank of A is zero exactly if A is nilpotent. This talk will present some results and methods related to the enumeration of certain classes of nilpotent matrices over finite fields and adapt them to the case of specified stable rank (with additional properties). The key ingredient is an adaptation of an algorithm due to Crabb (2006) which involves nothing more than Gaussian elimination and is considerably more elementary than other approaches to these problems in the literature.

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Colouring exact distance graphs of chordal graphs

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MSC2000: 05C15, 05C12, 05C17

Given a graph $G = (V, E)$ and a positive integer p, define the *exact distance-p graph* $G^{[\natural p]}$ as the graph having vertex set V and with an edge between vertices x and y if and only if x and y have distance p (i.e., any shortest path joining x and y has p edges).

Recently, there has been an effort to obtain upper bounds on the chromatic number of exact distance-p graphs [2, 3, 4]. For p even, upper bounds on $\chi(G^{[\natural p]})$ need to involve $\Delta(G)$ even for the class of trees. However, for p odd the number is known to be bounded by a constant for many important classes of graphs. A result of Nešetřil and Ossona de Mendez [3] tells us that this is the case for every graph class with bounded expansion. Van den Heuvel *et al.* [2] reproved this result, and provided explicit upper bounds on $\chi(G^{[\natural p]})$ for many graph classes. Although these new upper bounds dramatically improve on previous ones, in most cases it is not clear how their dependence on the distance is close to having the right order.

Turning from the very general to the particular, we focus on the chromatic number of exact distance graphs of chordal graphs. Using techniques specific to this class of graphs, we obtain improved upper bounds both for even an odd distances. Regarding their dependence on the distance considered, our upper bounds are very close to having the right order, as a result of Bousquet *et al.* [1] attests.

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On maximal partial ovoids of the elliptic quadric $Q^{-}(5,q)$ of $PG(5,q)$

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A partial ovoid O of $Q^{-}(5,q)$ is a set of points of $Q^{-}(5,q)$ such that every line of this quadric shares at most one point with O. A partial ovoid is called maximal if it cannot be extended to a larger partial ovoid. Several authors have studied maximal partial ovoids of $Q^{-}(5,q)$, but a complete knowledge of them is still far away. In particular, the spectrum of their sizes is an open problem. Concerning such a problem, many results have been found through a computer search. But, apart from this kind of investigation, we have few theoretical constructions and only sporadic cardinalities. In this talk I show a new geometric construction, which works for every value of q. By this, we get a new class of maximal partial ovoids of $Q^{-}(5,q)$, together with a non-interrupted interval of new cardinalities. In broad outline, the construction is the following. Let $\mathcal{S}_3$ be a 3-dimensional subspace of $PG(5,q)$, meeting $Q^{-}(5,q)$ at an elliptic quadric E. There are exactly $q+1$ hyperplanes of $PG(5,q)$ through $\mathcal{S}_3$. Two of them are tangent to $Q^{-}(5,q)$ and meet this quadric at tangent cones. The others meet $Q^{-}(5,q)$ at non-singular quadrics. We get the mentioned new class of maximal partial ovoids by choosing suitable points of E and suitable points of the above tangents cones.

Block Patterns in Generalized Euler Permutations and the Generalized Cluster Method

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(This talk is based on joint work with Ran Pan.)

MSC2000: 05A05, 05A15

Goulden and Jackson introduced a very powerful method to study the distributions of certain consecutive patterns in permutations, words, and other combinatorial objects called the cluster method. There are a number of natural classes of combinatorial objects which start with either permutations or words and add additional restrictions. These includes up-down permutations, generalized Euler permutations, words with no consecutive repeated letters, Young tableaux, and non-backtracking random walks. We develop an extension of the cluster method which we call the *generalized cluster method* to study the distribution of certain consecutive patterns in such restricted combinatorial objects.

Homomorphisms of Strongly Regular Graphs

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MSC2000: 05C50

A homomorphism is an adjacency preserving map between the vertex sets of two graphs. A n-vertex, k-regular graph is *strongly regular*, with parameters (n, k, λ, μ), if there exist numbers λ and μ such that every pair of adjacent vertices share λ common neighbors and every pair of non-adjacent vertices share μ common neighbors. We prove that if G and H are primitive strongly regular graphs with the same parameters and φ is a homomorphism from G to H, then φ is either an isomorphism or a coloring (homomorphism to a complete subgraph). Moreover, any such coloring is optimal for G and its image is a maximum clique of H. Therefore, the only endomorphisms of a primitive strongly regular graph are automorphisms or colorings. This confirms and strengthens a conjecture of Peter Cameron and Priscila Kazanidis that all strongly regular graphs are cores or have complete cores. The proof of the result is elementary, mainly relying on linear algebraic techniques, but can also be viewed as relying on a straightforward application of complementary slackness to a pair of semidefinite programs defining the Lovász theta number. This talk is based on the paper `arXiv:160100969`.

Descent and peak polynomials

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(This talk is based on joint work with Alexander Diaz-Lopez, Pamela Harris, and Erik Insko.)

MSC2000: 05A15

A permutation $\pi = \pi_1 \ldots \pi_n$ in the symmetric group $\mathfrak{S}_n$ has *descent set*

$$\operatorname{Des} \pi = \{i \mid \pi_i > \pi_{i+1}\}.$$

Otherwise put, these are the indices of initial elements of consecutive 21 patterns. Given a set S of positive integers and $n > \max S$, the *descent polynomial* of S is the cardinality

$$d(S; n) = \#\{\pi \in \mathfrak{S}_n \mid \operatorname{Des} \pi = S\}.$$

It is easy to prove, using the Principle of Inclusion and Exclusion, that this is a polynomial in n. However, properties of this polynomial do not seem to have been studied much in the literature. The *peak set* of π is

$$\operatorname{Pea} \pi = \{i \mid \pi_{i-1} < \pi_i > \pi_{i+1}\}.$$

These are indices of middle elements of either consecutive 132 or 231 patterns. Recently Billey, Burdzy, and Sagan proved that

$$\#\{\pi \in \mathfrak{S}_n \mid \operatorname{Pea} \pi = S\} = p(S; n) \cdot 2^{n-\#S-1}$$

where $p(S; n)$ is a polynomial in n which they dubbed the *peak polynomial* of S. These polynomials have since received the attention of a number of researchers. In this talk we will compare and contrast these two polynomials talking about their degrees, coefficients when expanded in a basis of binomial coefficients, roots, and analogues in other Coxeter groups.

Optimal incomplete block designs

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MSC2000: 05B05

When it comes to the best experimental designs statisticians need the ones that give them the estimates with minimum possible variance. Towards the quest of these designs different criteria can be used based upon the average variance, the maximum variance or the volume of the confidence region. This discussion gives the construction of optimal incomplete block designs with nearly minimal number of observations with respect to D-optimality (minimizing the volume of the confidence region) and A-optimality (minimizing the average variance) criteria. A unified approach towards the use of the associated graphs has been employed.

Keywords: A-optimality; D-optimality; Concurrence graph; Levi graph; phase-transition

Erdős-Rényi Theory for asymmetric tournaments

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MSC2000: 05C80, 05C20, 08A35

For given two asymmetric graphs (i.e. graphs with only trivial automorphism), how can we compare them for their asymmetry? The first work which answers this question is by Erdős-Rényi [2]. For a given finite graph G, they defined the *asymmetry number* $A(G)$ of G as follows:

$$A(G) = \min\{|E(G)\Delta E(G')| \mid G' \text{ is a symmetric graph over } V(G)\}.$$

In [2], they proved that $A(G) \leq [\frac{n-1}{2}]$ for all G with n vertices. By using probabilistic method, they showed that there exists a graph G such that $A(G) \geq \frac{n}{2} - O(\sqrt{n \log n})$ for sufficiently large n. Moreover, it follows that almost all finite graphs are asymmetric.
On the other hand, Erdős-Rényi [2] also investigated the asymmetry of countable graphs. They showed that countable random graphs are almost surely isomorphic to the graph R (called the *random graph*) which is highly symmetric. This result implies a remarkable gap between finite and countable graphs. From their work, many properties of $\mathrm{Aut}(R)$ have been investigated, see Cameron's survey paper [1].
In this talk which is based on [4], we consider the extension of these results for tournaments. First, we define the *asymmetry number* $A(T)$ for a tournament T. Next, we provide an upper bound for $A(T)$ and, by using the probabilistic method ([3]), we prove that this bound is asymptotically best. Moreover, we show an observation of $\mathrm{Aut}(RT)$, where RT is the *random tournament*, a natural tournament-analogue of R.
If time permits, we also introduce our digraph-extension of Erdős-Rényi theory.

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The three edge theorem

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(This talk is based on joint work with Christian Reiher and Vojtěch Rödl.)

MSC2000: 05C35, 05C65, 05C80

For a k-uniform hypergraph F let $\mathrm{ex}(n, F)$ be the maximum number of edges of a k-uniform n-vertex hypergraph H which contains no copy of F. Determining or estimating $\mathrm{ex}(n, F)$ is a classical and central problem in extremal combinatorics. While for $k = 2$ this problem is well understood, due to the work of Turán and of Erdős and Stone, only very little is known for k-uniform hypergraphs for $k > 2$. We focus on the case when F is a k-uniform hypergraph with three edges on $k + 1$ vertices. Already this very innocent (and maybe somewhat particular looking) problem is still wide open even for $k = 3$.

We consider a variant of the problem where the large hypergraph H enjoys additional hereditary density conditions. Questions of this type were suggested by Erdős and Sós about 30 years ago. We show that every k-uniform hypergraph H with density $> 2^{1-k}$ with respect to every large collection of k-cliques induced by sets of $(k - 2)$-tuples contains a copy of F. The required density 2^{1-k} is best possible as higher order tournament constructions show.

Our result can be viewed as a common generalisation of the first extremal result in graph theory due to Mantel (when $k = 2$ and the hereditary density condition reduces to a normal density condition) and a recent result of Glebov, Král', and Volec (when $k = 3$ and large subsets of vertices of H induce a subhypergraph of density $> 1/4$). Our proof for arbitrary $k \geq 2$ utilises the regularity method for hypergraphs.

Stanley symmetric functions and the slide product on weak compositions

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(This talk is based on joint work with Sami Assaf.)

MSC2000: 05E05

We introduce the fundamental slide basis [1] of the polynomial ring. Our motivation is to study firstly the Stanley symmetric functions, introduced by R. Stanley [5] as a tool for enumerating reduced decompositions of permutations, and secondly the geometrically-important basis of Schubert polynomials, the stable limits of which are the Stanley symmetric functions. We prove that Schubert polynomials expand as a positive sum of fundamental slide polynomials, and that the stable limits of fundamental slide polynomials are the fundamental quasisymmetric functions introduced by I. Gessel [4].

We define an explicit statistic on permutations [1] that gives the precise point at which the fundamental slide expansion of a Schubert polynomial stabilizes, i.e., when the number of terms in its slide expansion is exactly the number of terms in the fundamental quasisymmetric expansion of the corresponding Stanley symmetric function. We similarly define an explicit statistic on weak compositions [2], giving the stability point for the fundamental slide expansion of a key polynomial. For vexillary permutations, i.e., those avoiding the pattern 2143, Schubert polynomials and key polynomials coincide.

The slide polynomial basis has positive structure constants, and we give a positive combinatorial Littlewood-Richardson rule for these numbers. As a main ingredient in our rule, we define the slide product on weak compositions [1], generalizing the shuffle product [3] on compositions. Using this and results above, we give a positive Littlewood-Richardson rule for the slide expansion of products of Schubert polynomials, and formulas for products of Stanley symmetric functions in terms of Schubert structure constants.

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The Davenport constant of finite abelian groups

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MSC2000: 11B75

Let G be a finite abelian group. Define the *sum* of a multiset $\{g_1, \ldots, g_n\}$ of elements $g_i \in G$ to be $g_1 + \cdots + g_n$. A *zero-sum free* multiset over G is a multiset of elements of G with no submultiset whose sum is equal to 0_G. The *Davenport constant* of G indicates the size of the largest zero-sum free multiset over G. Though determining the Davenport constant of a group is a combinatorial problem, it has interesting applications in Number Theory. More precisely, if R is the ring of integers of some algebraic number field with ideal class group isomorphic to G and α is an irreducible element in R, then the Davenport constant of G is the maximal number of prime ideals which occur in the prime ideal decomposition of the ideal aR. It is known that the Davenport constant of G is at least $1 + d^*(G)$ where $d^*(G)$ is a certain constant that is computed using the invariant factor decomposition of G. There was a conjecture that this bound is always tight, but counterexamples are now known for many groups G of rank 4 or more. However, the conjecture has been established for many classes of groups, in particular it is known that $D(G) = 1 + d^*(G)$ when G has rank at most 2. Whether the conjecture holds when G has rank 3 is still an open problem. In this talk I will review finite abelian groups for which the precise value of the Davenport constant has been found, present the value of the Davenport constant of the smallest finite abelian group of rank 3 for which the value was previously unknown, and present a new general polynomial upper bound on the Davenport constant of G in terms of $d^*(G)$. The general polynomial upper bound is quadratic in $d^*(G)$ and I will show how it improves to a linear polynomial in $d^*(G)$ if G is isomorphic to a specific infinite class of groups.

Distance Magic Labelings of Distance Regular Graphs

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(This talk is based on joint work with I Wayan Palton Anuwiksa.)

MSC2000: 05C78

For an arbitrary set of distances $D \subseteq \{0, 1, \ldots, diam(G)\}$, a graph G is said to be D-magic if there exists a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ and a constant k such that for any vertex x, $\sum_{y \in N_D(x)} (y) = k$, where $N_D(x) = \{y \in V(G) | d(x, y) \in D\}$.

We define a D-distance graph of a graph G, denoted by $\Delta_D(G)$, as the graph with vertex set $V(G)$ and edge set $\{\{x, y\} | d_G(x, y) \in D\}$. We shall search for D-magic distance regular graphs for various D by utilising the spectrums of G and $\Delta_D(G)$.

Thursday 2.35 Level 1 Auditorium

Regular and bi-rotary maps of negative prime Euler characteristic

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(Based on a joint work with Joint work with A. Breda and D. Catalano)

MSC2000: 05C10, 05C25, 57M15

Pseudo-orientable maps were introduced by Steve Wilson [3] to describe non-orientable maps with the property that opposite orientations can consistently be assigned to adjacent vertices. In this paper we classify the pseudo-orientable maps whose local-orientation-preserving automorphism group acts regularly on arcs – the *bi-rotary maps* – on surfaces of negative prime Euler characteristic. Unlike other classification results for highly symmetric maps on such surfaces (e.g. those in [1, 2] for regular maps) we do not use the powerful Gorenstein-Walter result on the structure of groups with dihedral Sylow 2-subgroups in our proofs.

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Inversions in random node labelings of random trees

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(This talk is based on joint work with Xing Shi Cai, Cecilia Holmgren, Svante Janson and Tony Johansson.)

MSC2000: 05C5

The notion of an inversion in a rooted labelled tree is a generalization of an inversion in a permutation. Say that u is an ancestor of v if the unique path from the root to v passes through u. Given node labelling π we have an inversion if $\pi(u) > \pi(v)$ and u is an ancestor of v. Thus the total number of inversions in a randomly labelled path rooted at one of its end points is the number of inversions in a random permutation.

We deduce the distribution of total number of inversions in randomly lablled b-ary trees as well as two random tree models: split trees and Galton-Watson trees. Our work strengthens previously existing results on the number of distributions in Galton-Watson trees.

Smallest snarks with oddness 4

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(This talk is based on joint work with Edita Máčajová.)

MSC2000: 05C15, 05C70

The oddness of a bridgeless cubic graph G is the smallest number of odd circuits in a 2-factor of G. Oddness is one of the most important invariants of snarks because several important conjectures in graph theory can be reduced to snarks of oddness 4 or larger. In this talk we deal with the problem of determining the smallest order of a nontrivial snark of oddness 4. (Here 'nontrivial' means girth at least 5 and cyclic connectivity at least 4.) We prove that the smallest order of a nontrivial snark with oddness 4 and cyclic connectivity 4 is 44, and characterise all snarks of order 44 with this property. The proof relies on a detailed analysis of 3-edge-colourings conflicting on a cycle-separating 4-edge-cut, an extensive computer search, and a closure theorem for cubic graphs with cyclic connectivity 4 due to Andersen, Fleischner and Jackson (1988).

Juxtaposing Catalan permutation classes with monotone ones

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(This talk is based on joint work with Robert Brignall.)

MSC2000: 05A05, 05A15, 05A19

Juxtapositions are a simple special case of permutation grid classes. Each permutation in a grid class can be drawn into a grid so that the subpermutation in each box is in the class specified by the corresponding cell in a gridding matrix. Grid classes found application mainly as tools to study the structure of other permutation classes, but because of their more general applicability, the study of grid classes in their own right has emerged. Exact enumeration being an example. However, the difficulty of handling multiple griddings: that is, enumerating 'griddable' objects rather than 'gridded' ones, has prevented progress.

As a first step towards enumerating more general grid classes, we replace one cell in the gridding matrix M of a monotone grid class by a Catalan class, that is, one avoiding a single permutation of length 3. For simplicity, we restrict our attention to 1×2 grids and present a clean and unified way to enumerate all such grid classes.

Cycle partitioning in hypergraphs

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(This talk is based on joint work with Sebastián Bustamante and Hiêp Hàn.)

MSC2000: 05C15, 05C65

Recent times have seen plenty of activity in the area of monochromatic partitioning. A result of Bessy and Thomassé, confirming a conjecture of Lehel, states that every complete graph whose edges are coloured with two colours contains two monochromatic cycles, of distinct colours, which together span all the vertices. We discuss extensions of this result to hypergraphs, considering tight cycles, loose cycles and l-cycles.

Monday 10.55 Main Auditorium

Optimal Ramp Schemes and Related Combinatorial Objects

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MSC2000: 05B15

In 1996, Jackson and Martin proved that a strong ideal ramp scheme is equivalent to an orthogonal array. However, there was no good characterization of ideal ramp schemes that are not strong. Here we show the equivalence of ideal ramp schemes to a new variant of orthogonal arrays that we term *augmented orthogonal arrays*. We give some constructions for these new kinds of arrays, and, as a consequence, we also provide parameter situations where ideal ramp schemes exist but strong ideal ramp schemes do not exist.

Plenary lecture

Thursday 9.00 Main Auditorium

Robustness of graph properties

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MSC2000: 05C35, 05D40

A typical result in graph theory says that a graph G, satisfying certain conditions, has some property $\mathcal{P}$. Once such a theorem is established, it is natural to ask how strongly G satisfies $\mathcal{P}$. Can one strengthen the result by showing that G possesses $\mathcal{P}$ in a robust way? What measures of robustness can one utilize? In this survey, we discuss various measures that can be used to study robustness of graph properties, illustrating them with examples.

On Antiadjacency Matrix Properties

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(This talk is based on joint work with Bevina D. Handari, Ferry Firmansyah, Nora Hariadi, Wildan.)

MSC2000: 05C50

Finite graph can be represented by several matrices such as adjacency matrix, Laplacian matrix,and incidence matrix. Let A be an adjacency matrix of a graph G. An antiadjacency matrix is defined as $B = J - A$ where J is the matrix with all entries equal to 1. In this talk we discuss antiadjacency matrix properties. The first property has relation with directed graph, especially directed acyclic graph. We show that the characteristic polynomial of antiadjacency matrix can give the information on the number of paths with certain length in a directed acyclic graph. Further, if we consider of characteristic polynomial of antiadjacency matrix of general directed graph, we can also obtain some information from the coefficient of the polynomial characteristic. The second property has relation with regular graph and digraph.

[1] R.B. Bapat, *Graphs and Matrices*, Hindustan Book Agency. Springer, 1-12, 2010.

[2] G. Gordon, E. McMahon, A Characteristic Polynomial for Rooted Graphs and Rooted Digraphs, *Discrete Mathematics*, 232(1-3):19-33, 2001.

[3] R.P. Stanley, A Matrix for Counting Paths in Acyclic Digraphs, *Journal of Combinatorial Theory*, Series A, 74(1):169-172, 1996.

[4] W. Qiu, W. Yan, The Coefficients of Laplacian Characteristic Polynomials of Graphs, *Linear Algebra and Its Applications*, 435:2474-2479, 2012.

Generating subgraphs in chordal graphs

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(This talk is based on joint work with Vadim E. Levit.)

MSC2000: 05C69

A graph G is *well-covered* if its maximal independent sets are of the same cardinality [6]. Assume that a weight function w is defined on its vertices. Then G is *w-well-covered* if all maximal independent sets are of the same weight. For every graph G, the set of weight functions w such that G is w-well-covered is a *vector space* [4], denoted $WCW(G)$ [3].

Let B be a complete bipartite induced subgraph of G on vertex sets of bipartition B_X and B_Y. Then B is *generating* if there exists an independent set S such that $S \cup B_X$ and $S \cup B_Y$ are both maximal independent sets of G. If $B \approx K_{1,1}$, then the unique edge in B is *relating*. Generating subgraphs play an important role in finding $WCW(G)$.

Deciding whether an input graph G is well-covered is **co-NP**-complete [5, 7]. Hence, finding $WCW(G)$ is **co-NP**-hard. Deciding whether an edge is relating is **NP**-complete. Therefore, deciding whether a subgraph is generating is **NP**-complete as well.

A graph is *chordal (triangulated)* if every induced cycle is a triangle [1]. It is known that $WCW(G)$ can be constructed polynomially if G is chordal [2]. Thus recognizing well-covered chordal graphs is a polynomial problem. We present a polynomial algorithm for recognizing relating edges and generating subgraphs in chordal graphs.

[1] C. Berge, Some classes of perfect graphs, in "*Graph Theory and Theoretical Physics*" (F. Harary, ed.) 155–166, Academic Press, New York, 1967.

[2] J. I. Brown and R. J. Nowakowski, Well covered vector spaces of graphs, *SIAM Journal on Discrete Mathematics* 19:952–965, 2006.

[3] J. I. Brown, R. J. Nowakowski and I. E. Zverovich, The structure of well-covered graphs with no cycles of length 4, *Discrete Mathematics* 307:2235-2245, 2007.

[4] Y. Caro, N. Ellingham and G. F. Ramey, Local structure when all maximal independent sets have equal weight, *SIAM Journal on Discrete Mathematics* 11:644-654, 1998.

[5] V. Chvatal and P. J. Slater, A note on well-covered graphs, Quo Vadis, Graph Theory?, *Annals of Discrete Mathematics*, North Holland, Amsterdam 55:179-182, 1993.

[6] M. D. Plummer, Some covering concepts in graphs, *Journal of Combinatorial Theory* 8:91-98, 1970.

[7] R. S. Sankaranarayana and L. K. Stewart, Complexity results for well-covered graphs, *Networks* 22:247-262, 1992.

On transversals in intercalated latin squares and in the Cayley table of the group $\mathbb{Z}_2^m$

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MSC2000: 05B15

A latin square L of order n is an $n \times n$-table filled by n symbols so that each symbol appears exactly once in each row and each column. Every latin square is the Cayley table of some quasigroup and vice versa.

A transversal in a latin square of order n is a set of n entries which includes exactly one entry from each row and column and one of each symbol. Let $T(L)$ denote a number of transversals in a latin square L.

A latin subsquare of order 2 is called an intercalate. We will say that a latin square of order n is intercalated if it is composed of $n^2/4$ pairwise disjoint intercalates.

Theorem 1. *Let L be an intercalated latin square of order n. Then the number of transversals in L is divisible by $2^{\frac{n}{2}+1}$. In particular, if a group of order n contains a subgroup of order 2, then the number of transversals in the Cayley table of the group is divisible by $2^{\frac{n}{2}+1}$.*

A question on divisibility of a number of transversals in the Cayley tables of groups was investigated, for example, in [1], and see [2] for a survey.

Let $\mathcal{L}_m$ denote the latin square of order $n = 2^m$ that is the Cayley table of the group $\mathbb{Z}_2^m$.

Theorem 2. *There exists a constant c independent of m such that*

$$T(\mathcal{L}_m) \geq c2^{m2^{m-2}} = cn^{n/4}.$$

[1] B. D. McKay, J. C. McLeod, and I. M. Wanless. The number of transversals in a Latin square. *Des. Codes Cryptogr.*, 40:269–284, 2006.

[2] I. M. Wanless. Transversals in latin squares: a survey. *Surveys in Combinatorics 2011, London Math. Soc. Lecture Note Ser.*, 392:403–437, 2011.

An upper bound on the size of diamond-free families of sets

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(This talk is based on joint work with Dániel Grósz and Abhishek Methuku.)

MSC2000: 05D05

Let $\mathrm{La}(n, P)$ be the maximum size of a family of subsets of $[n] = \{1, 2, \ldots, n\}$ not containing P as a (weak) subposet. The diamond poset, denoted Q_2, is defined on four elements x, y, z, w with the relations $x < y, z$ and $y, z < w$. $\mathrm{La}(n, P)$ has been studied for many posets; one of the major open problems is determining $\mathrm{La}(n, Q_2)$. It is conjectured that $\mathrm{La}(n, Q_2) = (2 + o(1))\binom{n}{\lfloor n/2 \rfloor}$, and infinitely many significantly different, asymptotically tight constructions are known.

Studying the average number of sets from a family of subsets of $[n]$ on a maximal chain in the Boolean lattice $2^{[n]}$ has been a fruitful method. We use a partitioning of the maximal chains and introduce an induction method to show that $\mathrm{La}(n, Q_2) \leq (2.20711 + o(1))\binom{n}{\lfloor n/2 \rfloor}$, improving on the earlier bound of $(2.25 + o(1))\binom{n}{\lfloor n/2 \rfloor}$ by Kramer, Martin and Young.

An improved lower bound for Folkman's Theorem

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(This talk is based on joint work with József Balogh, Sean Eberhard, Bhargav Narayanan and Adam Zsolt Wagner.)

MSC2000: 05D10, 05D40

Folkman's Theorem asserts that for each $k \in \mathbb{N}$, there exists a natural number $n = F(k)$ such that whenever the elements of $[n] := \{1, \ldots, n\}$ are two-coloured, there exists a set $A \subset [n]$ of size k with the property that all the sums of the form $\sum_{x \in B} x$, where B is a nonempty subset of A, are contained in $[n]$ and have the same colour. In 1989, Erdős and Spencer showed that $F(k) \geq 2^{ck^2/\log k}$, where $c > 0$ is an absolute constant. In this talk we describe how one can improve this bound significantly by showing that $F(k) \geq 2^{2^{k-1}/k}$ for all $k \in \mathbb{N}$.

Finding Balance: Split Graphs and Related Classes

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(This talk is based on joint work with Karen Collins.)

MSC2000: 05C17

A graph is a split graph if its vertex set can be partitioned into a clique and an independent set. A split graph is unbalanced if there exist two such partitions that are distinct. Cheng, Collins and Trenk (2016), discovered the following interesting counting fact: unlabeled, unbalanced split graphs on n vertices can be placed into a bijection with all unlabeled split graphs on $n-1$ or fewer vertices. In this talk we translate these concepts and the theorem to different combinatorial settings.

Contractibility of Graphs

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(This talk is based on joint work with Michael B. Smyth.)

MSC2000: 05C75

Homomorphism graphs are graphs whose vertices are homomorphisms. A graph is said to be contractible if its induced homomorphism graph is connected. In this work we study properties of graph contractibility.

On strict-double-bound numbers of graphs and cut sets

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(This talk is based on joint work with K. Ikeda, K. Ogawa, S. Tashiro and S. Tagusari.)

MSC2000: 05C62, 05C76

For a poset $P = (X, \leq_P)$, the *strict-double-bound graph* of P is the graph $\mathrm{sDB}(P)$ on $V(\mathrm{sDB}(P)) = X$ for which vertices u and v of $\mathrm{sDB}(P)$ are adjacent if and only if $u \neq v$ and there exist elements $x, y \in X$ distinct from u and v such that $x \leq_P u \leq_P y$ and $x \leq_P v \leq_P y$. The *strict-double-bound number* $\zeta(G)$ of a graph G is defined as $\min\{\ n\ ;\ \mathrm{sDB}(P) \cong G \cup \overline{K}_n$ for some poset $P\}$.

In [1] Scott gave some upper bound of this number. In [2] we gave an upper bound of strict-double-bound numbers of non-trivial trees. In [2], [3] and [4] we also obtained results on the sum operation.

We obtain an upper bound of strict-double-bound numbers of graphs with cutsets to induce complete subgraphs.

Theorem 1. *Let G be a graph with a cutset S, where the induced subgraph $\langle S\rangle_V$ is a complete subgraph and $G - S$ has components $G_1, G_2, ..., G_k$. For $i = 1, 2, ..., k$, let $H_i = \langle V(G_i) \cup S\rangle_V$ and $P(H_i)$ be a poset such that $\mathrm{sDB}(P(H_i)) \cong H_i \cup \overline{K}_{\zeta(H_i)}$. Then $\zeta(G) \leq \sum_{i=1}^{k} \zeta(H_i) - \sum_{i=1}^{k} |\operatorname{Min}(P(H_i); S)| - \sum_{i=1}^{k} |\operatorname{NoMin}(P(H_i); S)| + \max\{|\operatorname{Min}(P(H_i); S)|\ ; i = 1, 2, ..., k\} + \max\{|\operatorname{NoMin}(P(H_i); S)|\ ; i = 1, 2, ..., k\}$.*

Further we estimate upper bounds of strict-double-bound numbers of chordal graphs and k-trees.

[1] D. D. Scott, The competition-common enemy graph of a digraph, *Discrete Applied Mathematics* , **17** (1987), 269-280.

[2] S. Konishi, K. Ogawa, S. Tagusari and M. Tsuchiya, *Note on strict-double-bound numbers of paths, cycles, and wheels*, JCMCC, **83** (2012), 205-210.

[3] K. Ogawa, R. Soejima, S. Tagusari and M. Tsuchiya, Note on strict-double-bound numbers of nearly complete graphs missing some edges, *Discrete Mathematics*, **312** (2012), 584-587.

[4] K. Ogawa, S. Tagusari and M. Tsuchiya, Note on strict-double-bound graphs and numbers, *AKCE International Journal of Graphs and Combinatorics*, **11** (2014),127-13.

Diregularity of extremal networks

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(This talk is based on joint work with Grahame Erskine.)

MSC2000: 05C35

An important topic in extremal graph theory is the topology of networks that are optimal subject to certain restraints on the degree, order, diameter or other related parameters. The best known such problems are the degree/diameter problem and the degree/girth problem. In the directed setting, a fundamental question is: are extremal networks diregular? In this talk I will review previous work in this field and present some new results on diregularity of extremal networks. In particular, I will explain recent advances in our understanding of the diregularity of $(d, k, +\epsilon)$-digraphs, i.e. k-geodetic digraphs with minimum out-degree d and order $M(d, k) + \epsilon$, where $M(d, k)$ is the Moore bound for degree d and diameter k and $\epsilon > 0$ is the (small) excess of the digraph.

The equidistribution of some descent set based statistics on words

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MSC2000: 05A05

For a length n permutation π, $\mathsf{Des}\,\pi$ (respectively, $\mathsf{Desrc}\,\pi$) denotes the descent set of π (respectively, the set $\{n-i\,|\,i \in \mathsf{Des}\,\pi\}$, i.e, the descent set of the reverse-complement of π), and $\mathsf{Ides}\,\pi$ denotes the descent set of π^{-1}; and Des, Desrc and Ides become set valued statistics. In 1976 Foata and Schützenberger shown that the bistatistics $(\mathsf{Des}, \mathsf{Ides})$ and $(\mathsf{Desrc}, \mathsf{Ides})$ have the same distribution on the set of same length permutations. They proof uses the Robinson correspondence between permutations and ordered pairs of standard Young tableaux, and they asked for a proof that could avoid the use of that correspondence. In this presentations such a proof is given, and extending Ides to words we show that $(\mathsf{Des}, \mathsf{Ides})$ and $(\mathsf{Desrc}, \mathsf{Ides})$ have the same distribution on the set of rearrangements of the symbols of a word.

As a consequence, we show the joint equidistribution on the rearrangements of the symbols of a word of stat, maj and Ides, and of maj, stat and Ides, together with other statistics; here maj is the celebrated major index statistic, and stat is the generalization given by Kitaev and the present author (2016) of a Mahonian statistic which is defined originally on permutations in terms of vincular patterns by Babson and Steingrímsson (2000). This equidistribution generalizes from permutations to words a previous result due to Burnstein (2010) and on which our construction is based, and refines another one stated in the above mentioned 2016 paper.

Monday 3.30 Auditorium C

Improper Colourings inspired by Hadwiger's Conjecture

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(This talk is based on joint work with David R. Wood.)

MSC2000: 05C15

Hadwiger's Conjecture (1943) asserts that every graph without the complete graph K_t as a minor has a proper vertex-colouring using at most $t-1$ colours. Since the conjecture is stubbornly refusing to be proved, we might look at relaxed versions of it.

In this talk we relax the conclusion of the conjecture by considering two types of improper colourings for K_t-minor-free graphs: (1) colourings in which each monochromatic component has small degree, and (2) colourings in which each monochromatic component has small size. In both cases our new results greatly improve the existing results on these colourings. The proofs are based on an elementary decomposition result for graphs without K_t-minor that might be of independent interest.

[1] H. Hadwiger. Über eine Klassifikation der Streckenkomplexe. *Vierteljschr. Naturforsch. Ges. Zürich*, 88(2):133–142, 1943.

[2] Jan van den Heuvel and David R. Wood. Improper colourings inspired by Hadwiger's Conjecture. **arXiv:1704.06536 [math.CO]**, 2017.

Partial latin squares that embed in an infinite group but not into any finite group

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(This talk is based on joint work with Heiko Dietrich.)

MSC2000: 05B15, 05E15, 20D60

Informally, a partial latin square (PLS) P embeds in a group G if you can find a copy of P in the Cayley table of G. We can think of P as a set of triples $P \subseteq Y_1 \times Y_2 \times Y_3$ where Y_1, Y_2, Y_3 are finite sets (the rows, columns and symbols respectively). We say that P embeds in G if there exist injective maps $\phi_i \colon Y_i \to G$ for $i = 1, 2, 3$ such that $\phi_1(y_1)\phi_2(y_2) = \phi_3(y_3)$ for each $(y_1, y_2, y_3) \in P$. We answer a question of Hirsch and Jackson, who asked for the cardinality of the smallest P that embeds in some infinite group but not into any finite group. It's not even obvious that any such P exists (though they gave an example). Our proof uses computations to answer questions about finitely presented groups which are known to be algorithmically undecidable in general.

[1] H. Dietrich and I. M. Wanless, Small partial Latin squares that embed in an infinite group but not into any finite group, *J. Symbolic Comput.*, to appear.

[2] R. Hirsch and M. Jackson, Undecidability of representability as binary relations. *J. Symbolic Logic* **77** (2012), 1211–1244.

[3] I. M. Wanless and B. S. Webb, Small partial Latin squares that cannot be embedded in a Cayley table. *Australas. J. Combin.* **67(2)** (2017), 352–363.

Countably infinite homogeneous STS

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(This talk is based on joint work with Daniel Horsley.)

MSC2000: 05B07

A structure S is called *homogeneous* if every isomorphism between finitely generated substructures of S extends to an automorphism of S. Thus homogeneity means that any local symmetry is global, and homogeneous structures are highly symmetric with very large automorphism groups.

We are interested in homogenous Steiner triple systems, but how you regard STS can affect what the substructures are, and so can affect properties such as homogeneity.

If an STS is considered as a first order *relational* structure with a ternary relation which holds for (x, y, z) if and only if $\{x, y, z\}$ is a block, then the substructures are partial STS. However, if an STS is considered as a first order *functional* structure with a binary function (Steiner quasi-group) $x \circ y = z$ if and only if $\{x, y, z\}$ is a block, then the substructures are subsystems.

The only homogeneous STS as a relational structure is the Fano plane, but the problem is more interesting for STS regarded as functional structures.

Here we present uncountably many new (countably infinite) homogeneous Steiner triple systems that mirror the Henson graphs.

Rational Dyck paths and factor-free Dyck words

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(This talk is based on joint work with Daniel Birmajer and Juan B. Gil.)

MSC2000: 05A15, 05A19

Motivated by independent results of Bizley and Duchon, we study rational Dyck paths and their subset of factor-free elements. We give a bijection between rational Dyck paths and regular Dyck paths with ascents colored by factor-free words. More precisely, we give a combinatorial proof of the following theorem:

Theorem 1. *Let $\alpha, \beta \in \mathbb{N}$ with $\gcd(\alpha, \beta) = 1$. There is a bijection between the set of rational Dyck paths with slope $\frac{\beta}{\alpha}$ of length $(\alpha+\beta)n$ and the set $\mathfrak{D}_n^{\Theta}(\alpha+\beta)$, where $\mathfrak{D}_n^{\Theta}(\alpha+\beta)$ represents the set of Dyck paths of semilength $(\alpha+\beta)n$ whose ascents have length a multiple of $\alpha+\beta$ and such that an ascent of length $(\alpha+\beta)j$ may be colored in θ_j different ways. Here θ_j denotes the number of factor-free words of length $(\alpha+\beta)j$.*

Our bijection leads to a new statistic based on the reducibility level of the paths for which we provide a corresponding formula.

Theorem 2. *The number $r_{n,k}$ of rational Dyck paths with slope $\frac{\beta}{\alpha}$ of length $(\alpha+\beta)n$ that have reducibility level equal to k is given by*

$$r_{n,k} = \binom{(\alpha+\beta)n}{k-1} \frac{(k-1)!}{n!} B_{n,k}(1!\theta_1, 2!\theta_2, \dots),$$

where $B_{n,k}(x_1, x_2, \dots)$ are partial Bell polynomials.

Time permitting, we will discuss alternative formulas for various enumerative sequences that appear in the context of rational Dyck paths.

Plenary lecture

Wednesday 11.15 Main Auditorium

Some Applications of Relative Entropy in Additive Combinatorics

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MSC2000: 11B30

In this talk we survey some recent applications of relative entropy in additive combinatorics. Specifically, we examine to what extent entropy-increment arguments can replace or even outperform more traditional energy-increment strategies or alternative approximation arguments based on the Hahn-Banach theorem, which have been instrumental in proving Szemerédi's theorem and the Green-Tao theorem on long arithmetic progressions in the primes.

Tutte Invariants for Alternating Dimaps

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(This talk is based on joint work with Graham Farr and Kerri Morgan.)

MSC2000: 05C99

An *alternating dimap*, which was introduced by Tutte in 1948, is an embedded Eulerian directed graph where the edges incident with each vertex are directed inwards and outwards alternately. Three reduction operations for alternating dimaps were investigated by Farr [1]. A *minor* of an alternating dimap can be obtained by reducing some of its edges using the reduction operations. Unlike classical minor operations, these reduction operations do not commute in general. A *Tutte invariant* for alternating dimaps is a function F defined on every alternating dimap such that F is invariant under isormorphism, multiplicative over components, and which obeys a linear recurrence relation involving reduction operations. We characterise the Tutte invariants for alternating dimaps introduced by Farr [1]. As a result of the non-commutativity of the reduction operations, the Tutte invariants are not always well defined. For some of the existing Tutte invariants, we investigate the properties of alternating dimaps that are required in order to obtain well defined Tutte invariants.

[1] G. E. Farr. Minors for alternating dimaps, preprint, 2013. http://arxiv.org/abs/1311.2783v3

All graphs with degree 0 or 4 whose complements have 4-cycle systems

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(This talk is based on joint work with Chin-Mei Fu.)

MSC2000: 05C70

A decomposition of a graph G is a collection $\mathcal{H} = \{H_1, H_2, \ldots, H_s\}$ of subgraphs of G such that $\bigcup_{i=1}^{s} E(H_i) = E(G)$ and $E(H_i) \cap E(H_j) = \emptyset$ for $1 \leq i < j \leq s$. If H_i is isomorphic to a graph H for $1 \leq i \leq s$, then we say that G has an *H-decomposition* or G can be decomposed into H. If H is isomorphic to a copy of C_k, k-cycle, then we say G has a k-cycle decomposition or G can be decomposed into k-cycles and $\mathcal{H}$ is a *k-cycle system* of G.

We call graph G 4-*sufficient* if each vertex of G is even degree and $|E(G)|$ is divisible by 4. Let H be a subgraph of K_n with degree 0 or 4 and $K_n - H$ be a graph which forms from K_n by removing the edges in H. Suppose the degree of each vertex of H is 0. A. Kotzig had proved that K_n is 4-sufficient if and only if K_n has a 4-cycle system. If the degree of each vertex of H is not all 0, then the induced subgraph of H by the vertices with degree 4 is a 4-regular subgraph of K_n. In this talk, we will show that any 4-sufficient graph $K_n - H$ has a 4-cycle system.

Generalized transversals, generalized vertex covers and node-fault-tolerance in graphs

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MSC2000: 05C35

The transversal number $\tau(G)$ of a hypergraph G is the minimum cardinality of a set of vertices that intersects all edges of G. For $r \geq 1$ define

$$f_r = \sup \frac{\tau(G)}{|V(G)| + |E(G)|},$$

where G ranges over all r-uniform hypergraphs. In 1990 Alon proved that as r tends to infinity

$$f_r = (1 + o(1))\frac{\ln r}{r}.$$

We consider the following generalization of this problem. Given a family $\mathcal{H}$ of hypergraphs, an $\mathcal{H}$-transversal of a hypergraph G is a set of vertices that intersects the vertex set of every subgraph H of G such that $H \in \mathcal{H}$. The $\mathcal{H}$-transversal number $\tau_{\mathcal{H}}(G)$ is the minimum cardinality of an $\mathcal{H}$-transversal of G. Define

$$f_r(\mathcal{H}) = \sup \frac{\tau_{\mathcal{H}}(G)}{|V(G)| + |E(G)|},$$

where G ranges over all r-uniform hypergraphs. Although the problem has been defined for all r's, in this talk we consider it only for graphs (i.e. $r = 2$). Consequently an $\mathcal{H}$-transversal will be called an $\mathcal{H}$-vertex cover. Note that if $\mathcal{H}$ is the family of cycles then an $\mathcal{H}$-vertex cover is a well known object called feedback vertex set, which has wide applications in operating systems, database systems etc. Furthermore, if $\mathcal{H} = \{P_k\}$, then an $\mathcal{H}$-vertex cover is called a k-path vertex cover, which has some important applications, as well.

We prove that

$$f_2(K_q) = \frac{1}{2q-1}, \quad f_2(P_3) = 1/4, \quad f_2(P_4) = 1/5, \quad f_2(C_4) = 1/6, \text{ and}$$
$$\frac{1}{5\sqrt{2q}+1} \leq f_2(C_q) \leq f_2(P_q) \leq 1/5 \text{ for } q \geq 5.$$

In order to prove these results we establish a connection between $\mathcal{H}$-transversals and the node-fault-tolerance in graphs. Furthermore, we derive some bounds on the $\mathcal{H}$-independence number.

Specifying a positive threshold function via extremal points

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(This talk is based on joint work with Vadim Lozin, Igor Razgon, Elena Zamaraeva, Nikolai Yu. Zolotykh.)

MSC2000: 68Q32

An extremal point of a positive threshold Boolean function f is either a maximal zero or a minimal one. It is known that if f depends on all its variables, then the set of its extremal points completely specifies f within the universe of threshold functions. However, in some cases, f can be specified by a smaller set. The minimum number of points in such a set is the specification number of f. It was shown in [2] that the specification number of a threshold function of n variables is at least $n+1$. In [1] it was proved that this bound is attained for nested functions and conjectured that for all other threshold functions the specification number is strictly greater than $n+1$. We resolve this conjecture negatively by exhibiting threshold Boolean functions of n variables, which are non-nested and for which the specification number is $n+1$. On the other hand, we show that the set of extremal points satisfies the statement of the conjecture, i.e. a positive threshold Boolean function depending on all its n variables has $n+1$ extremal points if and only if it is nested.

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Some bridges between lambda calculus and graphs on surfaces

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MSC2000: 05C30, 05C99

In recent years, studies of the combinatorics of lambda calculus have revealed some surprising connections to the theory of graphs on surfaces. In the talk, after a quick introduction to lambda calculus, I will give a brief survey of these enumerative connections [1, 2, 3, 4], then focus on the correspondence between rooted trivalent maps and linear lambda terms, explaining how it may be seen as encoding a Tutte decomposition of trivalent maps with boundary.

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Bounds of the sizes of families with uncomplemented and mutually incomparable conditions

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(This talk is based on joint work with J. Goldwasser, A. J. W. Hilton)

MSC2000: 05D05

Let $[n]$ denote the finite set $\{1, 2, \cdots, n\}$. For $A \subseteq [n]$, define $\bar{A} = [n] \setminus A$. A set family $\mathcal{A}$ consisting of subsets of $[n]$ is called *uncomplemented* if for each $A \in \mathcal{A}$, $\bar{A} \notin \mathcal{A}$. Two subsets A, B of $[n]$ are called *strongly incomparable* if $A \neq B$ and neither properly contains the other. They are called *weakly incomparable* if neither properly contains the other. In 1976, A. J. W. Hilton proved that if $\mathcal{A}$ and $\mathcal{B}$ are two uncomplemented, mutually strongly incomparable families of subsets of $[n]$, then $|\mathcal{A}| + |\mathcal{B}| \leqslant 2^{n-1}$. In this talk, we show that, Hilton's result can be generalized to the case of three families and the bound 2^{n-1} is still best possible. Moreover, we prove that for k set families $\mathcal{A}_1, \cdots ,, \mathcal{A}_k$ on $[n]$, the following two statements are equivalent.

1. If $\mathcal{A}_1, \cdots, \mathcal{A}_k$ are uncomplemented, mutually weakly incomparable families on $[n]$, then
$$|\mathcal{A}_1| + \cdots + |\mathcal{A}_k| \leqslant max(2^{n-1}, k\binom{n}{\lfloor \frac{n}{2} \rfloor + 1}).$$

2. If $\mathcal{A}_1, \cdots, \mathcal{A}_k$ are uncomplemented, mutually strongly incomparable families on $[n]$, then
$$|\mathcal{A}_1| + \cdots + |\mathcal{A}_k| \leqslant 2^{n-1}.$$

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Shuffle-Compatible Permutation Statistics

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(This talk is based on joint work with Ira M. Gessel.)

MSC2000: 05A05, 05E05, 16T30

This talk is a summary of the first in-depth investigation of "shuffle-compatible" permutation statistics. Given a length m permutation π and a length n permutation σ on disjoint sets of integers, we say that a length $m+n$ permutation τ is a *shuffle* of π and σ if π and σ are subsequences of τ. The set of shuffles of π and σ is denoted $S(\pi, \sigma)$. For example, $S(53, 16) = \{5316, 5136, 5163, 1653, 1536, 1563\}$.

We call a permutation statistic st *shuffle-compatible* if for disjoint permutations π and σ, the multiset $\{\, \mathrm{st}(\tau) \mid \tau \in S(\pi, \sigma) \,\}$—that is, the distribution of the statistic st among all shuffles of π and σ—depends only on $\mathrm{st}(\pi)$, $\mathrm{st}(\sigma)$, and the lengths of π and σ. Our definition is motivated by Richard Stanley's theory of P-partitions [3], which implies the shuffle-compatibility of the following classical permutation statistics: the descent set Des, the descent number des, the major index maj, and the pair $(\mathrm{des}, \mathrm{maj})$.

In this talk, we develop a theory of shuffle-compatibility for descent statistics, which are permutation statistics that depend only on the descent set and length of a permutation. Every shuffle-compatible statistic st can be associated with an algebra whose multiplication can be thought of as algebraically encoding the shuffle-compatibility property of st; we call this the *shuffle algebra* of st. For example, the descent set shuffle algebra is isomorphic to the algebra of quasisymmetric functions QSym [2].

Our main results are as follows. We establish a necessary and sufficient condition for the shuffle-compatibility of a descent statistic, which also shows that the shuffle algebra of any shuffle-compatible descent statistic is isomorphic to a quotient algebra of QSym. We also give a dual version of this condition which exploits the duality between QSym and the coalgebra of noncommutative symmetric functions **Sym** [1]. These results are then used to show that several well-known descent statistics are shuffle-compatible and to characterize their shuffle algebras.

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